Scand. J. of Economics 108(1), 15–38, 2006
DOI: 10.1111/j.1467-9442.2006.00437.x

Monetary Policy, Expectations and Commitment*

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Abstract

Commitment in monetary policy leads to equilibria that are superior to those from optimal discretionary policies. A number of interest-rate reaction functions and instrument rules have been proposed to implement or approximate commitment policy. We assess these rules in terms of whether they lead to a rational expectations equilibrium that is both locally determinate and stable under adaptive learning by private agents. A reaction function that appropriately depends explicitly on private sector expectations performs particularly well on both counts.

Keywords: Commitment; interest-rate rule; learning; stability; determinacy
JEL classification: *E*52; *E*31; *D*84

I. Introduction

Many recent models of monetary policy emphasize the importance of forward-looking aspects of the economy, where expectations of private agents significantly influence the economic outcome. If expectations about the future are part of the equilibrating mechanisms in the economy, it is well known that standard intertemporal optimization of economic policy by the government is in general subject to the problem of time inconsistency, so

* This work was in part carried out when Honkapohja visited the European Central Bank's Directorate General Research as part of their Research Visitor Programme. Their hospitality and support are gratefully acknowledged. We thank, without implicating, Marco Bassetto, V. V. Chari, Charles Goodhart, Nobu Kiyotaki, Ed Nelson, Evi Pappa, Danny Quah, Lars E. O. Svensson, Mike Woodford and the ECB staff for useful comments. Financial support from the US National Science Foundation Grant No. 0136848 and from grants by the Academy of Finland, Bank of Finland, Yrjö Jahnsson Foundation and Nokia Group is gratefully acknowledged. The research was to a large extent done while Honkapohja was affiliated with the Research Unit on Economic Structures and Growth, University of Helsinki.

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that a policymaker has incentives to deviate, in later periods, from the optimal plan obtained in the first period. In contrast, discretionary policies are obtained through policy optimization separately in each period and are time consistent, but typically the resulting sequence of discretionary policy decisions will not lead to the overall intertemporal optimum. The losses from discretionary policies can be quantitatively significant, and this has provided an impetus for finding ways to achieve the optimum or at least to improve the outcome.

While earlier papers on time consistency focused on the inflation bias in monetary policy, recent work has shown that even if inflation bias does not arise under appropriate goals of the policymaker, the issue of commitment vs. discretion still obtains. Discretion leads to what is called a "stabilization bias" and there are gains to commitment; see Clarida, Gali and Gertler (1999), McCallum and Nelson (2004), Svensson and Woodford (2005) and Woodford (1999a, 1999b).

Woodford (1999a, 1999b) suggests that monetary policymaking should be based on the *timeless perspective*. This concept is a rule-based policy that is obtained by respecting the optimality conditions from the full intertemporal optimization under commitment, except for the current decision-making period. In other words, according to Woodford (1999a, p. 293), the policymaker follows "the pattern of behavior to which it would have wished to commit itself at a date far in the past". The gains from committing to this policy, relative to the discretionary policies, can be significant; see McCallum and Nelson (2004). In this paper we adopt the timeless perspective formulation and refer to the corresponding optimal monetary policy as the "commitment solution".

Most of the recent literature on monetary policy, including all of the references above, has been conducted under the hypothesis of rational expectations (RE). However, this may not be an innocuous assumption, as shown by Bullard and Mitra (2002) and Evans and Honkapohja (2003b). The assumption of RE should not be taken for granted, since expectations can be out of equilibrium, at least for a period of time, as a result of exogenous events such as structural shifts in the economy. Economic policies should be designed to avoid instabilities that can arise from expectational errors and the corrective behavior of economic agents in the face of such errors.

The issue of temporary errors in forecasting, and the consequent correction mechanisms, have been widely studied in recent research using the adaptive learning approach.[1] Bullard and Mitra (2002) consider the stability of equilibria when monetary policy is conducted using some variant of the

[1] Evans and Honkapohja (2001) is a treatise on adaptive learning and its macroeconomic implications. Evans and Honkapohja (1999), Marimon (1997) and Sargent (1993) are surveys of the field.

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Taylor interest-rate rule and argue that monetary policymaking should take into account the constraints on the policy parameters implied by learnability. Evans and Honkapohja (2003b) show that certain standard forms of optimal discretionary interest-rate setting by the central bank lead to instability as economic agents unsuccessfully try to correct their forecast functions over time, so that the economy fails to converge to the desired rational expectations equilibrium (REE). We there propose an alternative way to implement optimal discretionary policy that always leads to stability under learning.[2]

The research on adaptive learning and monetary policy has so far focused on the performance of discretionary optimal policies or ad hoc interest-rate rules. This paper takes up optimal policy under commitment and studies whether this facilitates convergence of private expectations to the optimal REE. On intuitive grounds one might think that commitment favors stability under learning by leading to more forecastable dynamics than when policy is re-optimized every period. We argue that while this can indeed be the case, stability depends critically on the way the policy is implemented. Certain standard forms of central bank reaction functions do not or do not always provide stability under learning. However, there is another implementation, depending explicitly on private expectations, that always performs well in this respect.

A related concern addressed by Bernanke and Woodford (1997), Svensson and Woodford (2005), Woodford (1999b) and others is that it is desirable for policy rules to yield determinacy, i.e., locally unique REE, to ensure that there are no nearby suboptimal REE. We show that for all parameter values, our proposed "expectations-based" rule satisfies the dual criteria of determinacy and stability under learning.

II. The Model

We use a linearized model that is very common in the literature; see Clarida *et al*. (1999) for this particular formulation and references to the literature. The original non-linear framework is based on a representative consumer, a continuum of firms producing differentiated goods under monopolistic competition and subject to constraints on the frequency of price changes, as originally suggested by Calvo (1983).

The behavior of the private sector is described by two equations:

$$x_t = -\varphi(i_t - E_t^* \pi_{t+1}) + E_t^* x_{t+1} + g_t, \tag{1}$$

which is the "IS" curve derived from the Euler equation for consumer optimization, and

[2] The literature on learning and monetary policy is surveyed in Evans and Honkapohja (2003a).

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$$\pi_t = \lambda x_t + \beta E_t^* \pi_{t+1} + u_t, \tag{2}$$

which is the price-setting rule for the monopolistically competitive firms. The interpretation of (1) and (2) is discussed further in the Appendix on temporary equilibrium.

Here x_t and π_t denote the output gap and inflation for period t, respectively. i_t is the nominal interest rate, expressed as the deviation from the steady-state real interest rate. The determination of i_t is discussed below. $E_t^* x_{t+1}$ and $E_t^* \pi_{t+1}$ denote the private sector expectations of the output gap and inflation next period. Since our focus is on learning behavior, these expectations need not be rational (E_t without * denotes RE). The parameters φ and λ are positive and β is the discount factor so that $0 < \beta < 1$.

The shocks g_t and u_t are assumed to be observable and follow

$$\begin{pmatrix} g_t \\ u_t \end{pmatrix} = F \begin{pmatrix} g_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} \tilde{g}_t \\ \tilde{u}_t \end{pmatrix}, \quad \text{where } F = \begin{pmatrix} \mu & 0 \\ 0 & \rho \end{pmatrix}, \tag{3}$$

$0 < |\mu|, |\rho| < 1$, and $\tilde{g}_t \sim iid(0, \sigma_g^2)$, $\tilde{u}_t \sim iid(0, \sigma_u^2)$ are independent white noise. g_t represents shocks to government purchases and/or potential output. u_t represents any cost-push shocks to marginal costs other than those entering through x_t.[3] The u_t shock is important for policy issues since the g_t shock can be fully offset by appropriate interest-rate setting. μ and ρ are assumed known (if not, they could be estimated).

Assume RE for the moment. Monetary policy is derived from minimization of a quadratic loss function:

$$E_t \sum_{s=0}^{\infty} \beta^s \left(\pi_{t+s}^2 + \alpha x_{t+s}^2 \right). \tag{4}$$

This type of optimal policy is often called "flexible inflation targeting" in the current literature; see e.g. Svensson (1999, 2003). α is the relative weight on the output target and pure inflation targeting would be the case $\alpha = 0$. Note that, first, the policymaker is assumed to have the same discount factor as the private sector and, second, the target value of the output gap is set at zero, implying that the classical problem of inflation bias does not arise. For brevity, the inflation target is also set at zero (introducing non-zero targets would not change the conclusions of our analysis). We treat the

[3] For possible interpretations of the u_t shock, see Clarida *et al*. (1999), Erceg, Henderson and Levin (2000) and Woodford (2003, Ch. 6).

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policymaker's preferences as exogenously given. It is also well known, as in e.g. Rotemberg and Woodford (1999), that the quadratic loss function (4) can be viewed as an approximation of the utility function of the representative consumer.[4]

The full intertemporal optimum under RE, usually called the commitment solution, is obtained by maximizing (4) subject to (2) for all periods t, $t + 1$, $t + 2$, ... The first-order conditions are written as

$$\lambda\pi_t = -\alpha x_t \tag{5}$$

$$\lambda\pi_{t+s} = -\alpha(x_{t+s} - x_{t+s-1}), \tag{6}$$

for $s = 1, 2, 3 \ldots$ The time inconsistency of the commitment solution is evident from (5), since this places a requirement that is specific to the current period and is different from the corresponding requirement (6) for later periods.

As noted in the introduction, the timeless perspective resolution to the problem of the time inconsistency of optimal policy is that the policymaker should respect the optimality conditions above, except for the current period when the optimization is done. In our context this amounts to using (6) also for the current period (and neglecting (5)). This yields the *commitment optimality condition*:[5]

$$\lambda\pi_t = -\alpha(x_t - x_{t-1}). \tag{7}$$

We remark that (7) is sometimes called a "specific targeting rule" in the literature.

We next compute the REE of interest. It can be shown that the dynamic system in x_t and π_t defined by (2) and (7) has a unique non-explosive RE solution. This solution can be expressed as a linear function of the state variables x_{t-1} and u_t and is known as the "minimal state variable" (MSV) solution; see McCallum (1983). It is obtained by using the method of undetermined coefficients, expressing the REE as

$$x_t = b_x x_{t-1} + c_x u_t \tag{8}$$

$$\pi_t = b_\pi x_{t-1} + c_\pi u_t. \tag{9}$$

As shown by McCallum and Nelson (2004), imposing RE implies that b_x must satisfy

[4] Like much of the literature on monetary policy, we do not explicitly introduce the budget constraint of the government. This is justified by assuming that fiscal policy is set "passively" in the sense of Leeper (1991) and ensures that the intertemporal budget constraint of the government is satisfied.

[5] Clarida *et al*. (1999, p. 1681), McCallum and Nelson (2004), Woodford (1999b, Sec. 3.1) and Woodford (1999a, Appendix) present this optimality condition.

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$$\beta b_x^2 - \gamma b_x + 1 = 0,$$

where $\gamma = 1 + \beta + \lambda^2/\alpha$. This has two solutions, but the one of interest is[6]

$$\bar{b}_x = (2\beta)^{-1}[\gamma - (\gamma^2 - 4\beta)^{1/2}]. \tag{10}$$

This delivers a stationary REE for all values of structural parameters, since $0 < \bar{b}_x < 1$, and corresponds to the policy optimum. The other coefficients are

$$\bar{b}_\pi = (\alpha/\lambda)(1-\bar{b}_x), \quad \bar{c}_x = -[\lambda + \beta\bar{b}_\pi + (1-\beta\rho)(\alpha/\lambda)]^{-1}, \quad \bar{c}_\pi = -(\alpha/\lambda)\bar{c}_x.$$

We refer to this REE as the *optimal REE*.

III. Optimal Interest-rate Setting

Thus far we have formulated the concept of optimal monetary policy under RE and reviewed the derivation of the corresponding REE using the existing literature. This derivation did not rely on the aggregate demand curve (1), which depends on the interest rate and which can be used to determine the interest rate that implements the desired optimal equilibrium. Computation of the appropriate interest rate leads to a functional relationship that is often called a *reaction function*, since the optimality condition (7) will be exactly met. Interest-rate rules that respond to endogenous and exogenous variables, but do not respect (7), are instead called *instrument rules*; we analyze some instrument rules below in Section V.[7]

As has become apparent from the earlier literature, interest-rate setting in the form of a reaction function can be implemented in different ways depending on what is assumed to be known in the policy optimization. In this paper we consider several possibilities, extending the analysis in Evans and Honkapohja (2003b) for discretionary policy. For each reaction function we test its performance in two ways.

First, we determine if the resulting REE is determinate, i.e., that it is the unique stationary REE under the reaction function. If a solution is indeterminate, there exist other stationary RE solutions nearby and, as is well known, these can include a dependence on extraneous variables or "sunspots". Second, we determine whether the REE corresponding to the reaction function implementing optimal policy is stable under adaptive learning by private agents. Here we formally analyze whether the RE solution is

[6] The other root for b_x is always larger than one and therefore generates explosive time paths.
[7] Our terminology largely agrees with that of Svensson (2003) and Svensson and Woodford (2005). They call the optimality condition (7) a "specific targeting rule" and the setting of the interest rate instrument, with (7) satisfied, a "reaction function" of the policymaker.

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E-stable, since E-stability is known to determine whether the solution is locally stable if private agents update their forecasts using least squares or closely related learning schemes. We remark that these are independent criteria. Our aim is to look for reaction functions for the interest rate that induce both determinacy and stability under learning.

The Fundamentals-based Reaction Function

A possible interest-rate rule to implement the optimal REE is obtained by computing

$$E_t\pi_{t+1} = \bar{b}_\pi \bar{b}_x x_{t-1} + (\bar{b}_\pi \bar{c}_x + \bar{c}_\pi \rho)u_t$$
$$E_t x_{t+1} = \bar{b}_x^2 x_{t-1} + (\bar{b}_x + \rho)\bar{c}_x u_t,$$

inserting these expectations and (8) into (1), and solving for the interest rate:

$$i_t = \psi_x x_{t-1} + \psi_g g_t + \psi_u u_t, \tag{11}$$

where

$$\psi_x = \bar{b}_x[\varphi^{-1}(\bar{b}_x - 1) + \bar{b}_\pi]$$
$$\psi_g = \varphi^{-1}$$
$$\psi_u = [\bar{b}_\pi + \varphi^{-1}(\bar{b}_x + \rho - 1)]\bar{c}_x + \bar{c}_\pi \rho.$$

We refer to (11) as the *fundamentals-based reaction function*, since its derivation is based solely on models (1) and (2), the optimality condition (7) and the assumption that the economy is in a stationary REE. The corresponding reaction function under discretion is identical, except that $\psi_x = 0$. Comparing discretion to (11) we see that the former is an open-loop policy whereas (11) has a feedback from lagged endogenous variables.

We emphasize that the derivation of this interest-rate rule presupposes RE on the part of both the private agents and the policymaker. The dependence on a lagged output gap reflects the commitment aspect of the optimal policy. We note that interest-rate setting according to (11) is quite similar to the "reaction functions" in equation (2.30) in Svensson and Woodford (2005) and equation (3.5) in Svensson (2003). Their models differ from the model in this paper, but the setting of interest rates according to lagged output and observable exogenous variables is the key common feature for their setups and (11).[8]

We now analyze the model with interest-rate rule (11) for determinacy and stability under learning. For this purpose, combining (1), (2) and (11),

[8] Our model does not include the unobservable judgement variables that are introduced in Svensson (2003) to capture further model uncertainties.

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we write the reduced form of the model in terms of general (possibly non-rational) expectations as

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} 1 & \varphi \\ \lambda & \beta + \lambda\varphi \end{pmatrix} \begin{pmatrix} E_t^* x_{t+1} \\ E_t^* \pi_{t+1} \end{pmatrix} + \begin{pmatrix} -\varphi\psi_x & 0 \\ -\lambda\varphi\psi_x & 0 \end{pmatrix} \begin{pmatrix} x_{t-1} \\ \pi_{t-1} \end{pmatrix} + \begin{pmatrix} -\varphi\psi_u \\ 1 - \lambda\varphi\psi_u \end{pmatrix} u_t \,. \tag{12}$$

Does the Fundamentals-based Reaction Function Yield Determinacy?

To analyze determinacy, we apply well-known methodology; see e.g. the Appendix to Chapter 10 in Evans and Honkapohja (2001). The basic steps are to rewrite the model in first-order form and to compare the number of non-predetermined variables with the number of eigenvalues of the forward-looking matrix that lie inside the unit circle. When these numbers are equal, the model is determinate and has a unique non-explosive solution. Intuitively, each root inside the unit circle provides a side condition that ties down one non-predetermined variable. If there are fewer eigenvalues inside the unit circle than non-predetermined variables, then the model is indeterminate and there exist multiple non-explosive solutions. In particular, in the indeterminate case there exist multiple stationary solutions that depend on sunspot variables. In contrast to the optimal REE, these other REE will not satisfy (7), the necessary conditions for an optimum.[9]

Whether the determinacy condition holds depends on the structural parameters:

Proposition 1. *Under the fundamentals-based reaction function there are parameter regions in which the model is determinate and other regions in which it is indeterminate.*

As an illustration we consider three different calibrations found in the literature.[10]

Calibration W: $\beta = 0.99$, $\varphi = (0.157)^{-1}$ and $\lambda = 0.024$.

[9] Other stationary REE that satisfy (2) cannot satisfy (7) because, as previously noted, the system (2) and (7) has a unique stationary RE solution.

[10] The calibrations on both Clarida, Gali and Gertler (2000) and Woodford (1999b) are for quarterly data. However, Woodford (1999b) uses quarterly interest rates and measures inflation as quarterly changes in the log price level, while Clarida *et al*. (2000) use annualized rates for both variables. We adopt the Woodford measurement convention, and therefore our CGG calibration divides by 4 both the σ and κ values reported by Clarida *et al*. (2000).

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Calibration CGG: $\beta = 0.99$, $\varphi = 4$ and $\lambda = 0.075$.

Calibration MN: $\beta = 0.99$, $\varphi = 0.164$ and $\lambda = 0.3$.

These are taken, respectively, from Woodford (1999b), Clarida *et al*. (2000) and McCallum and Nelson (1999). Straightforward numerical calculations show that for small values of α the steady state is indeterminate, while for larger values of α the model is determinate. (With the calibrated parameter values the borderlines are approximately $\alpha = 0.16$, 0.47 and 278, for the three calibrations.) Determinacy thus arises only for some values of α. The domain of values for α that gives determinacy depends sensitively on the calibration but, in general, sufficient flexibility in inflation targeting is needed to ensure determinacy of equilibrium under the reaction function (11).

We remark that we are treating α as a free policy preference parameter as is often done in the applied literature. If instead (4) is obtained as an approximation to the welfare of the representative consumer, α, φ and λ all depend on deep preference and price-setting parameters. Because there are more than three deep structural parameters, however, there are degrees of freedom for α given β, φ and λ.[11]

Learning Instability with the Fundamentals-based Reaction Function

Derivation of the interest-rate reaction function (11) presupposed that economic agents in the model have RE. However, suppose now that private agents have possibly non-rational expectations, which they try to correct through adaptive learning. We assume that the policymaker does not explicitly take this private agent learning into account, and continues to set policy according to (11). We are thus analyzing whether, under (11), the optimal REE is robust to transient errors in forecasting by private agents.

We apply the standard methodology of adaptive learning; see footnote 1 for references. The system under adaptive learning, more specifically under least squares learning, and stability of an REE under learning are formulated as follows.

The central assumption is that at each period t private agents have a perceived law of motion (PLM) that they use to make forecasts. In vector notation the PLM is

$$y_t = a_t + b_t y_{t-1} + c_t v_t, \quad \text{where } y_t = \begin{pmatrix} x_t \\ \pi_t \end{pmatrix}, v_t = \begin{pmatrix} g_t \\ u_t \end{pmatrix}. \qquad (13)$$

[11] In Rotemberg and Woodford (1999), φ is determined by a parameter of the utility function for aggregate consumption. α and λ depend on this and two other preference parameters as well as independent price-setting parameters. Analysis of the feasible range of (α, φ, λ) would require a separate study.

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The parameters (a_t, b_t, c_t) are updated over time using least squares. (This updating might, for example, be done by an econometric forecasting firm that supplies forecasts to the agents.) Note that for the reduced form (12) the optimal REE can be written as

$$y_t = \bar{a} + \bar{b}y_{t-1} + \bar{c}v_t,$$

where $\bar{a} = 0$ and where the second column of $\bar{b}$ is zero. The PLM (13) has the same form as this REE but, in general, (a_t, b_t, c_t) need not equal the REE values $(0, \bar{b}, \bar{c})$.

Given the PLM and the current value of v_t, the forecast functions of the private agents are $E_t^* y_{t+1} = a_t + b_t E_t^* y_t + c_t E_t^* v_{t+1}$ or

$$E_t^* y_{t+1} = a_t + b_t(a_t + b_t y_{t-1} + c_t v_t) + c_t F v_t, \tag{14}$$

where (a_t, b_t, c_t) are the parameter values of the forecast functions that agents have estimated on the basis of past data up to and including period $t - 1$. Note that we are assuming that current exogenous variables, and lagged but not current endogenous variables, are in the information set when forecasts are made. This is in line with much of the literature. At certain points in the text we consider an alternative information assumption in which expectations depend on current endogenous variables.

These forecasts are used in decisions for period t, which yields the temporary equilibrium, also called the actual law of motion (ALM), for $y_t = (x_t, \pi_t)$ with the given PLM. The temporary equilibrium or ALM provides a new data point and agents are then assumed to re-estimate the parameters (a_t, b_t, c_t) with data through period t and use the updated forecast functions for period $t + 1$ decisions. Along with v_{t+1} these in turn yield the temporary equilibrium for period $t + 1$ and the learning dynamics continues with the same steps in subsequent periods. The REE $(0, \bar{b}, \bar{c})$ is said to be *stable under learning* if the sequence (a_t, b_t, c_t) converges to $(0, \bar{b}, \bar{c})$ over time.

The Appendix gives the stability conditions for convergence to an REE under least squares learning. The central idea is to obtain a mapping T from the PLM parameters (a, b, c) to the implied ALM parameters, $T(a, b, c)$. The REE corresponds to a fixed point of this map and a stability condition, known as E-stability, can be defined in terms of a differential equation describing partial adjustment of the PLM parameters towards the ALM parameters. E-stability turns out to provide the conditions for stability of an REE under least squares and closely related learning rules.

In earlier work, we showed that discretionary policy, using interest-rate setting based on fundamentals, leads to instability because learning by private agents fails to lead the economy to the REE corresponding to the optimal policy without commitment; see Evans and Honkapohja (2003b). It

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would seem possible that the full commitment policy implemented with (11) might perform better than discretion in this respect, because of the feedback of the output gap on interest rates. However, we have:

Proposition 2. *The fundamentals-based reaction function leads to instability under learning for all structural parameter values.*

The proof is given in the Appendix. The source of instability lies in the interaction between the IS curve (1) and the price-setting curve (2). Some intuition is obtained by considering a PLM (a, b, c) in which all parameters are held fixed at the optimal REE values, except for the inflation intercept term a_π. In this case the mapping from PLM to ALM becomes one-dimensional and takes the form

$$T_{a_\pi}(a_\pi) = constant + (\beta + \lambda\varphi)a_\pi.$$

Since β is close to one, for most parameter values we have $\beta + \lambda\varphi > 1$. a_π will therefore tend to be adjusted away from the equilibrium value. Intuitively, $a_\pi > 0$ corresponds to an exogenous positive shock to inflation expectations. This directly increases inflation by β times the shock. In addition, via (1) the inflation expectations shock lowers the real interest rate, thereby increasing output by φ times the shock, and through (2) this raises inflation indirectly by $\lambda\varphi$ times the shock. These revisions of expected inflation toward actual inflation lead to a cumulative movement away from equilibrium. Under least squares learning the dynamics are, of course, much more complicated and in particular all of the parameters (a, b, c) adjust to forecast errors. The proof of Proposition 2 shows that under the fundamentals-based interest-rate policy, the system is always locally unstable, even in the case $\beta + \lambda\varphi < 1$.[12]

In Evans and Honkapohja (2004) we illustrate the instability result by a simulation that shows an explosive path for the inflation rate that emerges after about 110 periods. Other simulations for the fundamentals-based rule show a variety of unstable paths. Of course, faced with such a path, the policymaker would alter the policy rule and private agents would also be motivated to alter their learning rule. However, such simulations illustrate the stability problems inherent in the fundamentals-based rule: under this policy rule the economy will be subject to expectational instability.

[12] An interesting question is whether instrument rules of the form $i_t = \psi_x x_{t-1} + \psi_g g_t + \psi_u u_t$ yield unstable REE under learning even when the coefficients are not chosen to deliver the optimal reaction function. It can be shown that stable (and determinate) cases do exist if $1 - \beta^2 - \lambda\varphi\beta > 0$.

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In summary, under private agent learning, the policymaker's ability to commit to optimal policies is not sufficient to stabilize the economy, if the policy reaction function is based on observable exogenous shocks and the lagged output gap in the way suggested by the standard theory for optimal policy. We emphasize that under the fundamentals-based rule, the problem of instability arises even if the optimal REE is determinate.

An Expectations-based Reaction Function

The computation above deriving the fundamentals-based reaction function relied heavily on the assumption that the economy is in the optimal REE. We now obtain a different interest-rate reaction function, under optimal policy, which does not make direct use of the RE assumption. Recognizing the possibility that private agents may have non-rational expectations during the learning transition, the policy rule is obtained by combining the optimality condition, the price-setting equation and the IS curve, for given private expectations. This leads to a policy rule in which interest rates depend on observed private expectations as well as on fundamentals. We call this rule the *expectations-based reaction function*.

Formally, combine the price-setting equation (2) and the optimality condition (7), while treating private expectations as given. This leads to

$$x_t = \frac{\lambda}{\alpha + \lambda^2}\left[\frac{\alpha}{\lambda}x_{t-1} - \beta E_t^* \pi_{t+1} - u_t\right].$$

Next, substitute this expression into the IS curve (1) and solve for i_t. This yields the expectations-based reaction function for interest-rate setting:

$$i_t = \delta_L x_{t-1} + \delta_\pi E_t^* \pi_{t+1} + \delta_x E_t^* x_{t+1} + \delta_g g_t + \delta_u u_t, \tag{15}$$

where

$$\delta_L = \frac{-\alpha}{\varphi(\alpha + \lambda^2)}, \quad \delta_\pi = 1 + \frac{\lambda\beta}{\varphi(\alpha + \lambda^2)}, \quad \delta_x = \delta_g = \varphi^{-1}, \quad \delta_u = \frac{\lambda}{\varphi(\alpha + \lambda^2)}.$$

Looking at the rule (15), it can be seen that its coefficients stipulate a relatively large response to expected inflation ($\delta_\pi > 1$) and that effects coming from the expected output gap and the aggregate demand shock are fully neutralized ($\delta_x = \delta_g = \varphi^{-1}$). The positive coefficients on private expectations are crucial for ensuring stability of the REE and the sizes of the coefficients are chosen so that the economy is led to the optimal REE.

We now consider determinacy and stability under learning for the expectations-based reaction function (15). The reduced form of the economy under (15) is

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$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} 0 & -\dfrac{\lambda\beta}{\alpha+\lambda^2} \\ 0 & \dfrac{\alpha\beta}{\alpha+\lambda^2} \end{pmatrix} \begin{pmatrix} E_t^* x_{t+1} \\ E_t^* \pi_{t+1} \end{pmatrix} + \begin{pmatrix} \dfrac{\alpha}{\alpha+\lambda^2} & 0 \\ \dfrac{\alpha\lambda}{\alpha+\lambda^2} & 0 \end{pmatrix} \begin{pmatrix} x_{t-1} \\ \pi_{t-1} \end{pmatrix} + \begin{pmatrix} -\dfrac{\lambda}{\alpha+\lambda^2} \\ \dfrac{\alpha}{\alpha+\lambda^2} \end{pmatrix} u_t. \tag{16}$$

It is clearly a desirable property of our proposed monetary policy rule that it does not permit the existence of other suboptimal stationary REE. However, as we have seen in the case of the fundamentals-based reaction function, having a determinate REE does not always ensure that it is attainable under learning. To analyze stability under learning we can again use the general matrix framework in the Appendix. As in the preceding section we endow private agents with the PLM, compute the corresponding forecast function and substitute them into (16). This yields the temporary equilibrium or ALM and we can study whether least squares learning converges to the REE under the expectations-based reaction function (15) by computing E-stability conditions.

The next proposition shows that our interest-rate rule performs well (see the Appendix for the formal proof).

Proposition 3. *Under the expectations-based reaction function (15), the optimal REE is both determinate and stable under learning for all structural parameter values.*

The key to our stability results is that monetary authorities raise interest rates, *ceteris paribus*, in response to increases in inflation and output forecasts by private agents, and lower interest rates in response to decreases in private expectations. Some intuition can be gained from the reduced form (16). An increase in inflation expectations now leads to an increase in actual inflation that is smaller than the change in expectations since $\alpha\beta/(\alpha + \lambda^2) < 1$. This dampened effect arises from the interest-rate reaction to changes in $E_t^*\pi_{t+1}$ and is a crucial element of the stability result.

The results of Proposition 3 can also be viewed in a different way. Under (15), the optimality condition (7) is satisfied for all possible expectations. Thus, the reduced form (16) is obtainable directly from (7) and (2), and it is a corollary that the specific targeting rule (7), advocated e.g. by Svensson (2003), is determinate and stable under learning for all parameter values. The point of Proposition 3 is precisely to show how implementation of (7) can be achieved using an interest-rate reaction function.

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In summary, Proposition 3 provides a remarkably strong result: the reaction function (15) passes both of the performance tests we earlier set forth. These positive results show that our analysis of optimal discretionary policy in Evans and Honkapohja (2003b) can be extended to implement optimal policy with commitment.

IV. Discussion

Thus far we have treated expectations as determined before the current values of endogenous variables are realized, as is evident from (14). This would be natural if agents obtain these forecasts from an econometric forecasting firm prior to entering the market-place. We briefly consider an alternative possibility that allows forecasts to be functions also of the current values of endogenous variables, so that

$$E_t^* y_{t+1} = a_t + b_t y_t + c_t F v_t.$$

This means that current decisions and forecasts of the agents are determined simultaneously. Private agents now have to be regarded as entering the market-place with the most recent estimates of the *forecast functions* (obtained from the forecasting firm), which are incorporated into the consumption and pricing plans. We remark that this stronger information assumption gives additional scope to monetary policy, since changes in interest rates will also have an immediate indirect effect on expectations.

Indeterminacy under the fundamentals-based reaction function is, of course, not affected since this is a property of the model under RE. Stability under learning can, in general, be affected by the alternative information assumption. It turns out that under the alternative information assumption and the fundamentals-based reaction function, there are parameter regions in which the model is stable under learning and other parameter regions in which it is unstable under learning. Instability arises for sufficiently small values of α. For example, for the MN calibration the borderline is approximately $\alpha = 1.830$. In contrast, we continue to have stability under the expectations-based reaction function; see Evans and Honkapohja (2004) for details and the proof.

Proposition 4. *Under the alternative information assumption and the expectations-based reaction function (15), the optimal REE is stable under learning for all structural parameter values.*

Several further points should be made concerning our results. First, although we have demonstrated our results in the context of least squares learning, the stability results will obtain under various generalizations of

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least squares.[13] In fact, the stability results for the expectations-based reaction function hold even for some forecast rules that do not converge to RE. This is true, for example, if private agents forecast both output and prices using simple adaptive expectations rules.

There are two potential limitations to implementing policy using our expectations-based rule.[14] One limitation is that high-quality contemporaneous observations of expectations may not be available. One possible way of dealing with this problem would be to construct proxies for private expectations. The central bank might then use forecasts based on recursive VARs, i.e., apply the same procedure that we are assuming is used by private agents. This procedure can achieve convergence under plausible auxiliary assumptions even if their priors differ. A second limitation is the assumption that the coefficients of the structural model (1) and (2) are known to the policymaker. With discretionary policy, it was shown in Evans and Honkapohja (2003b) that for an expectations-based policy implemented using estimated structural parameters, the REE remains locally stable under simultaneous learning by private agents and policymakers. An analogous argument can be made in the current case of optimal policy with commitment.

We remark that Jensen and McCallum (2002) have recently shown that modifying the optimality condition (7) to $\lambda\pi_t = -\alpha(x_t - \beta x_{t-1})$ appears to improve the policy performance, because it partially compensates for the timeless perspective neglect of the first-period optimality condition. Fundamentals- and expectations-based reaction functions can be derived which correspond to this modified optimality condition. It can be shown that our stability and instability results remain unchanged.

V. Alternative Policy Rules

The fundamentals-based rule (11) is specified in terms of lagged output. We might now ask whether stability can be achieved if the rule were expressed in terms of the lagged price level. The commitment optimality condition (7) can be written as $\lambda(p_t - p_{t-1}) = -\alpha(x_t - x_{t-1})$, where p_t is the log of the price level. This will be satisfied if $x_t = -(\lambda/\alpha)p_t + k$, for any constant k. It can be verified that the optimal REE satisfies

$$p_t = \bar{b}_x p_{t-1} + \bar{c}_p u_t + \bar{a}_p$$
$$x_t = \bar{b}_p p_{t-1} + \bar{c}_x u_t + \bar{a}_x,$$

[13] See e.g. the weighting schemes in Marcet and Sargent (1989) and inertial behavior in Evans, Honkapohja and Marimon (2001).

[14] For further discussion and formal details, see Evans and Honkapohja (2003a).

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for appropriate parameters with $\bar{b}_x$ as before. Following the earlier procedures, we can obtain an alternative fundamentals-based reaction function of the form

$$i_t = \eta_p p_{t-1} + \varphi^{-1} g_t + \eta_u u_t + \eta_0.$$

It can be shown that the optimal REE leads to instability when $\varphi > (\lambda/\alpha)$.[15]

Above we chose our recommended rule carefully to ensure both determinacy and stability under learning for all parameter values. In the literature alternative interest-rate rules have appeared, which can be interpreted as expectations-based reaction functions but which do not meet our tests. Consider the interest-rate reaction function

$$i_t = \left(1 - \frac{\lambda}{\alpha\varphi}\right) E_t \pi_{t+1} + \varphi^{-1} g_t,$$

suggested in Clarida *et al*. (1999, Sec. 4.2.2). Replacing $E_t\pi_{t+1}$ with $E_t^*\pi_{t+1}$ leads to a policy reaction function based in part on observed expectations. This policy rule is consistent with the optimal policy under commitment under the RE assumption. However, this reaction function can lead to indeterminacy and, furthermore, if $\beta + \lambda^2/\alpha > 1$ the optimal REE is not stable under learning.

McCallum and Nelson (2004) have recently suggested that, in place of interest-rate setting by a reaction function satisfying the optimality condition (7), there are well-performing instrument rules that can approximate (7). These instrument rules specify that the interest rate is moved towards a specified target value in response to deviations from commitment optimality. To begin, consider instrument rules of the form

$$i_t = \pi_t + \theta[\pi_t + (\alpha/\lambda)(x_t - x_{t-1})], \quad \theta > 0. \tag{17}$$

We call this the *approximate targeting rule*. Numerical results (details are given in the working paper version) indicate that, under (17), the steady state seems to be determinate and stable under learning for all values of α and θ.

As pointed out by McCallum and Nelson (2004), a difficulty with the approximate targeting rule (17) is that it presupposes that the policymaker can observe current output gap and inflation when setting i_t. If neither x_t nor π_t are observable at t, they find that a forward-looking version performs best under RE, e.g.

$$i_t = \tilde{E}_t\pi_{t+1} + \theta[\tilde{E}_t\pi_{t+1} + (\alpha/\lambda)(\tilde{E}_t x_{t+1} - \tilde{E}_t x_t)], \tag{18}$$

where $\tilde{E}_t(\cdot)$ denotes the expectations of the policymaker. Suppose that the expectations of the policymaker are formed like those of private agents.

[15] Formal details are available in the working paper version, cf. Evans and Honkapohja (2004).

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Table 1. *Critical values $\hat{\theta}$, $\bar{\theta}$ for indeterminacy and instability*

α	0.01	0.05	0.1	0.5	1.0	2.0
Indeterminacy	1.643	0.365	0.185	0.037	0.019	0.009
Instability	3.495	0.413	0.197	0.038	0.019	0.009

Determinacy and learnability for the rule (18) now depend on the values of the parameters. As an illustration we consider the CGG calibration. Determinacy obtains for sufficiently small values of the reaction parameter θ, but larger values $\theta > \hat{\theta}$ lead to indeterminacy. Correspondingly, learning stability obtains for sufficiently small values of θ, while larger values $\theta > \bar{\theta}$ can destabilize the economy. The boundaries $\hat{\theta}$ and $\bar{\theta}$ depend on the model parameters and, in particular, on the degree of flexibility α in inflation targeting. This is illustrated in Table 1. We remark that for $\hat{\theta} < \theta < \bar{\theta}$ we have stability but indeterminacy. Restricting θ to be relatively small to achieve stability is problematic since, under RE, rules with a small value of θ imply that deviations from optimality lead to only small corrections towards meeting the optimality condition. In some cases, the welfare losses can be substantial when θ is restricted to values consistent with stability.[16]

VI. Concluding Remarks

In this paper we have analyzed determinacy and stability under learning for alternative interest-rate reaction functions that aim to implement optimal monetary policy under commitment. Determinacy is desirable because it implies that there do not exist other (non-optimal) REE near the solution of interest. Stability under learning is desirable because it indicates that if private agents follow least squares learning they will converge over time to the optimal REE. These are independent criteria, as is evident from our results in Sections III and V.

Our analysis leads to the conclusion that the two desiderata are met by a policy that sets interest rates according to our expectations-based reaction function. In this monetary policy reaction function, interest rates respond to private expectations as well as to fundamentals, i.e., exogenous shocks and the lagged output gap. This interest-rate reaction function unambiguously delivers both determinacy and stability under learning for the economy, with the economy converging over time to the optimal REE.

In contrast, the fundamentals-based formulation does not perform well and problems with both indeterminacy and instability under learning arise.

[16] For further details see Evans and Honkapohja (2003a, 2004).

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The dependence on the lagged output gap implied by commitment is not sufficient to guarantee convergence under learning when interest-rate setting is based solely on fundamentals.

We also discussed alternative policy rules that aim to implement optimal policy. With the exception of the approximate targeting rule using contemporaneous information on inflation and output gap, which is questionable from the viewpoint of operationality, all of the alternatives examined had problems with determinacy and stability under learning. More generally, we reiterate that in monetary policy design, expectations have to be treated as subject to deviations from rational expectations with private agents following a natural econometric forecasting procedure. Optimal policy should be designed so that under private agent learning the economy is guided to the REE.

Appendix

Stability under Learning, General Methodology

Temporary Equilibrium. The starting point for models of adaptive learning is that agents have less information than is presumed under RE. Instead, private agents optimize using subjective (possibly non-rational) probability distributions over future variables. Given these subjective distributions, the standard Euler equations provide necessary conditions for optimal decisions, and we assume that the Euler equations for the current period specify the behavioral rule that gives current decisions as functions of the expected state next period. These Euler equations are then supplemented by rules for forecasting next period's values of the state variables. Thus, given their forecasts, agents make decisions for the current period according to the Euler equations. This kind of behavior is boundedly rational but, in our view, reasonable, since agents are attempting to make optimal decisions based on a perceived law of motion for the state variables.[17]

For the model at hand we provide a detailed discussion making use of the general equilibrium framework presented in Woodford (1996). Although we maintain the representative agent assumption, so that agents have identical expectations and make the same decisions, it will be useful to let i index individual firms or households. Consider first the Phillips curve (2). Let $\mathcal{P}_t^i$ be the price set by those firms that can do so, P_t the average price index and $\hat{\mathcal{P}}_t^i$ the deviation of the relative price $\mathcal{P}_t^i/P_t$ from its stationary value. Woodford shows that $\hat{\mathcal{P}}_t^i$ can be expressed as a linear function of current output and discounted sums of expected future outputs and inflations. This derivation can be viewed as using subjective expectations that need not be rational. Assuming that the law of iterated expectations holds at the level of the individual

[17] Recently, Preston (2005) has studied standard instrument rules for monetary policy when agents have a different behavioral rule in which long-horizon forecasts matter. The E-stability conditions appear to be unchanged; see Honkapohja, Mitra and Evans (2002) for further discussion.

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agent, quasi-differencing allows us to express $\hat{\mathcal{P}}_t^i$ as a linear function of $E_t^{i*}\hat{\mathcal{P}}_{t+1}^i$, x_t and $E_t^{i*}\pi_{t+1}$, where, of course, E_t^{i*} denotes the expectation of firm i. However, $\hat{\mathcal{P}}_{t+1}^i = \hat{\mathcal{P}}_{t+1}$ and there is a proportional relationship between inflation and $\hat{\mathcal{P}}_t$. Firms will thus observe from the data that $\hat{\mathcal{P}}_t^i$ is exactly proportional to π_t, and thus $\hat{\mathcal{P}}_t^i$ can be rewritten as a linear function of expected inflation $E_t^{i*}\pi_{t+1}$ and current output x_t.[18] This defines the optimal price-setting schedule for $\mathcal{P}_t^i$, as a function of P_t, x_t and $E_t^{i*}\pi_{t+1}$, which firms take to the market-place. In addition, we allow for an exogenous shock u_t to the price-setting schedule. In the temporary equilibrium, with identical firms and homogeneous forecasts, and using again the relationship between $\hat{\mathcal{P}}_t$ and inflation, we obtain (2).

Consider next the IS curve (1). The linearized Euler equation, which is standard, is given by $c_t^i = E_t^{i*}c_{t+1}^i - \varphi\left(i_t - E_t^{i*}\pi_{t+1}\right)$.[19] (Government purchases are assumed to enter the utility function in an additively separable way.) Although c_{t+1}^i will be determined by the household itself, a forecast is required to determine its optimal current consumption. Let ξ_t denote the proportion of government purchases in GDP, and let $\hat{\xi}_t = -\ln(1-\xi_t)$. From market clearing $c_t^i = c_t = x_t - \hat{\xi}_t$. Assume households observe from past data that $c_t^i = x_t - \hat{\xi}_t$ and make use of this relationship for forecasting their future consumption. For convenience, assume that $\hat{\xi}_t$ follows a known AR(1) process $\hat{\xi}_t = \mu\hat{\xi}_{t-1} + \tilde{\xi}_t$. Then $E_t^{i*}c_{t+1}^i = E_t^{i*}x_{t+1} - E_t^{i*}\xi_{t+1}$, which leads to the consumption schedule

$$c_t^i = E_t^{i*}x_{t+1} - \varphi(i_t - E_t^{i*}\pi_{t+1}) - \mu\hat{\xi}_t$$

submitted to the market-place. Assuming that the government comes to the market-place with its plan to purchase proportion ξ_t of output, we obtain (1), where $g_t = (1-\mu)\hat{\xi}_t$, in temporary equilibrium with identical households and homogeneous forecasts.

Given private expectations, these schedules along with the monetary policy rule determine a temporary equilibrium according to (1) and (2). Thus the values of π_t, x_t and i_t are simultaneously determined through market clearing, in the usual way, by the pricing and consumption schedules. To complete the description of the temporary equilibrium, we have to specify how expectations are formed. The main case considered in the text assumes that expectations are functions only of lagged endogenous variables and observable current shocks and are thus predetermined when the plans are brought to the market. This would be natural if forecasts were obtained from an econometric forecasting firm before going to the market-place. In the alternative information assumption in Section IV, the agents instead obtain forecasting functions from the firm and plug in observations of current endogenous variables at the market-place, so that π_t, x_t, i_t and the forecasts are all determined simultaneously.

The temporary equilibrium for the current period provides a new data point for the agents. Given these new data, the forecast functions are updated at the start of the following period using least squares. The stability question is whether this kind of (adaptive) learning behavior converges over time to REE of interest.

[18] We make the simplifying assumption that potential output is constant so that output can be identified with x_t. This assumption is easily relaxed.
[19] See e.g. McCallum and Nelson (1999) or Woodford (1996).

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Stability Conditions. When agents adjust their forecast functions over time, the dynamics of the economy is mathematically specified by a stochastic recursive algorithm, which is a special type of non-linear time-varying stochastic system. The conditions for convergence of such dynamics are formally obtained from the local stability conditions of an associated ordinary differential equation.[20] The latter conditions are in turn governed by what are called *expectational or E-stability conditions*. Evans and Honkapohja (2001) provide an extensive analysis of adaptive learning and its implications in macroeconomics; see also the other references in footnote 1. In this paper we simply present the E-stability conditions for a general matrix model

$$\begin{aligned} y_t &= A + ME_t^* y_{t+1} + QE_t^* y_t + Ny_{t-1} + Pv_t \\ v_t &= Fv_{t-1} + \tilde{v}_t, \end{aligned} \tag{A1}$$

where $\tilde{v}_t$ is multivariate white noise. This setup is sufficiently general to cover all rules considered in the paper. Usually either $Q = 0$ or $N = 0$.

For (A1) with $N \neq 0$, the REE of interest take the form $y_t = \bar{a} + \bar{b}y_{t-1} + \bar{c}v_t$. To define E-stability we consider PLMs

$$y_t = a + by_{t-1} + cv_t.$$

Using the methods in Evans and Honkapohja (2001, Ch. 10), for (A1) the mapping from PLM to ALM is given by

$$T(a, b, c) = (A + (Q + M(I + b))a, Mb^2 + Qb + N, Qc + M(bc + cF) + P).$$

The E-stability conditions can be stated in terms of the derivative matrices

$$DT_a = Q + M(I + \bar{b}) \tag{A2}$$

$$DT_b = \bar{b}' \otimes M + I \otimes M\bar{b} + I \otimes Q \tag{A3}$$

$$DT_c = F' \otimes M + I \otimes M\bar{b} + I \otimes Q, \tag{A4}$$

where $\otimes$ denotes the Kronecker product and $\bar{b}$ denotes the REE value of b.

Remark A1. *The necessary and sufficient conditions for E-stability are that all eigenvalues of $DT_a - I$, $DT_b - I$ and $DT_c - I$ have negative real parts.*[21]

When $N = 0$, the MSV solution takes the form

$$y_t = a + hv_t,$$

where in the REE the coefficients satisfy $a = (M + Q)a$ and $h = MhF + Qh + P$. E-stability conditions now require that the eigenvalues of the matrices

$$\begin{aligned} DT_a - I &= M + Q - I \\ DT_h - I &= F' \otimes M + I \otimes Q - I \end{aligned}$$

have negative real parts.

[20] This approach was first exploited in a learning context by Marcet and Sargent (1989).
[21] We exclude the exceptional cases where one or more eigenvalue has zero real part.

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Derivations and Proofs

To assess determinacy we write the system as

$$\begin{pmatrix} x_t \\ \pi_t \\ x_t^L \end{pmatrix} = J \begin{pmatrix} x_{t+1} \\ \pi_{t+1} \\ x_{t+1}^L \end{pmatrix} + other, \tag{A5}$$

where $x_t^L \equiv x_{t-1}$. Since there is one predetermined variable, determinacy holds when J has two eigenvalues inside and one outside the unit circle.

Proof of Proposition 1. From the reduced form (12) we obtain

$$J = \begin{pmatrix} 0 & 0 & 1 \\ 0 & \beta & \lambda \\ (\varphi\psi_x)^{-1} & \psi_x^{-1} & -(\varphi\psi_x)^{-1} \end{pmatrix}.$$

Straightforward numerical calculations for the calibrated example show that two eigenvalues of J lie outside the unit circle, and one lies inside, for small values of α, so that the steady state is indeterminate, while for larger values of α exactly one root lies outside the unit circle, and the model is determinate. We remark that continuity of eigenvalues implies that both regions contain open sets of parameters.

Proof of Proposition 2. We apply the E-stability conditions above, when the general model (A1) takes the specific form (12). In this case $Q = 0$ and

$$M = \begin{pmatrix} 1 & \varphi \\ \lambda & \beta + \lambda\varphi \end{pmatrix}, \quad N = \begin{pmatrix} -\varphi\psi_x & 0 \\ -\lambda\varphi\psi_x & 0 \end{pmatrix} \quad \text{and} \quad P = \begin{pmatrix} 0 & -\varphi\psi_u \\ 0 & 1 - \lambda\varphi\psi_u \end{pmatrix}.$$

In the E-stability conditions (A2)–(A4), the condition for b is independent of the other variables, while the conditions for a and c are dependent on b but not on each other. Because of this recursive structure, a necessary condition for stability is that $DT_a - I$, evaluated at the REE, has eigenvalues with negative real parts. This condition is equivalent to $\mathrm{tr}(DT_a - I) < 0$ and $\det(DT_a - I) > 0$.

Using the notation $b = (b_{ij})$, $j = 1, 2$, and evaluating variables at the REE, we have $b_{11} = \bar{b}_x$, $b_{21} = \bar{b}_\pi$ and $b_{12} = b_{22} = 0$. The coefficient matrix for a in (A2) for the reduced form (12) has the explicit form

$$DT_a - I = \begin{pmatrix} \bar{b}_x + \varphi\bar{b}_\pi & \varphi \\ \lambda(\bar{b}_x + 1) + (\beta + \lambda\varphi)\bar{b}_\pi & (\beta + \lambda\varphi) - 1 \end{pmatrix}. \tag{A6}$$

The determinant of the coefficient matrix (A6) is $(\beta - 1)\bar{b}_x - \varphi\bar{b}_\pi - \lambda\varphi < 0$ since $0 < \beta < 1$ and λ, φ, $\bar{b}_x$ and $\bar{b}_\pi > 0$. The result follows.

Proof of Proposition 3. From the reduced form (16) we obtain

$$J = \begin{pmatrix} 0 & 0 & 1 \\ 0 & \beta & \lambda \\ 0 & \beta\lambda/\alpha & (\alpha + \lambda^2)/\alpha \end{pmatrix}.$$

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The roots of J are 0 and

$$(2\alpha)^{-1}\left(\alpha+\alpha\beta+\lambda^2\pm\sqrt{(\alpha+\alpha\beta+\lambda^2)^2-4\alpha^2\beta}\right).$$

The non-zero roots are real and positive, with one root less than one and the other root larger than one. Since exactly two roots are inside the unit circle, determinacy follows.

Turning to E-stability, we have

$$DT_b-I=\begin{pmatrix}\dfrac{-\beta\lambda\bar{b}_\pi}{\alpha+\lambda^2}-1 & \dfrac{-\beta\lambda\bar{b}_x}{\alpha+\lambda^2} & 0 & 0\\ \dfrac{\alpha\beta\bar{b}_\pi}{\alpha+\lambda^2} & \dfrac{\alpha\beta\bar{b}_x}{\alpha+\lambda^2}-1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix} \tag{A7}$$

$$DT_c-I=\begin{pmatrix}\dfrac{-\lambda\beta\bar{b}_\pi}{\alpha+\lambda^2}-1 & -\dfrac{\lambda\beta\rho}{\alpha+\lambda^2} & 0 & 0\\ \dfrac{\alpha\beta\bar{b}_\pi}{\alpha+\lambda^2} & \dfrac{\alpha\beta\rho}{\alpha+\lambda^2}-1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix} \tag{A8}$$

$$DT_a-I=\begin{pmatrix}\dfrac{-\beta\lambda\bar{b}_\pi}{\alpha+\lambda^2}-1 & \dfrac{-\beta\lambda}{\alpha+\lambda^2}\\ \dfrac{\alpha\beta\bar{b}_\pi}{\alpha+\lambda^2} & \dfrac{\alpha\beta}{\alpha+\lambda^2}-1\end{pmatrix}. \tag{A9}$$

DT_b-I has two eigenvalues equal to -1. The remaining two eigenvalues are those of the 2×2 matrix in the top left corner of DT_b-I. The trace of this 2×2 matrix is $(\alpha+\lambda^2)^{-1}(-\beta\lambda\bar{b}_\pi+\alpha\beta\bar{b}_x)-2$, which is negative since the only positive term is less than one. Its determinant is $(\alpha+\lambda^2)^{-1}(\beta\lambda\bar{b}_\pi-\alpha\beta\bar{b}_x)+1$, which is positive as the only negative term is less than one absolute value (since $\beta<1$ and $0<\bar{b}_x<1$). Thus, all of the eigenvalues of (A7) have negative real parts.

The matrix (A8) has two eigenvalues equal to -1 and the remaining two are those of the 2×2 matrix in the top left corner. The trace of this 2×2 matrix is $(\alpha+\lambda^2)^{-1}(-\beta\lambda\bar{b}_\pi+\alpha\beta\rho)-2$. The only positive term (if $\rho>0$) is less than one and so the trace is always negative. (If $\rho<0$, all terms are negative.) Its determinant is $(\alpha+\lambda^2)^{-1}(\beta\lambda\bar{b}_\pi-\alpha\beta\rho)+1$ and the only (possibly) negative term is less than one, so the determinant is positive. Thus DT_c-I is a stable matrix. Finally, we note that the top left 2×2 matrix with $\rho=1$ is identical to the matrix (A9), so that the latter is also a stable matrix.

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First version submitted December 2004;
final version received April 2005.

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