Expectations, Learning and Macroeconomic Policy

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Lecture 4

Liquidity traps, learning and stagnation

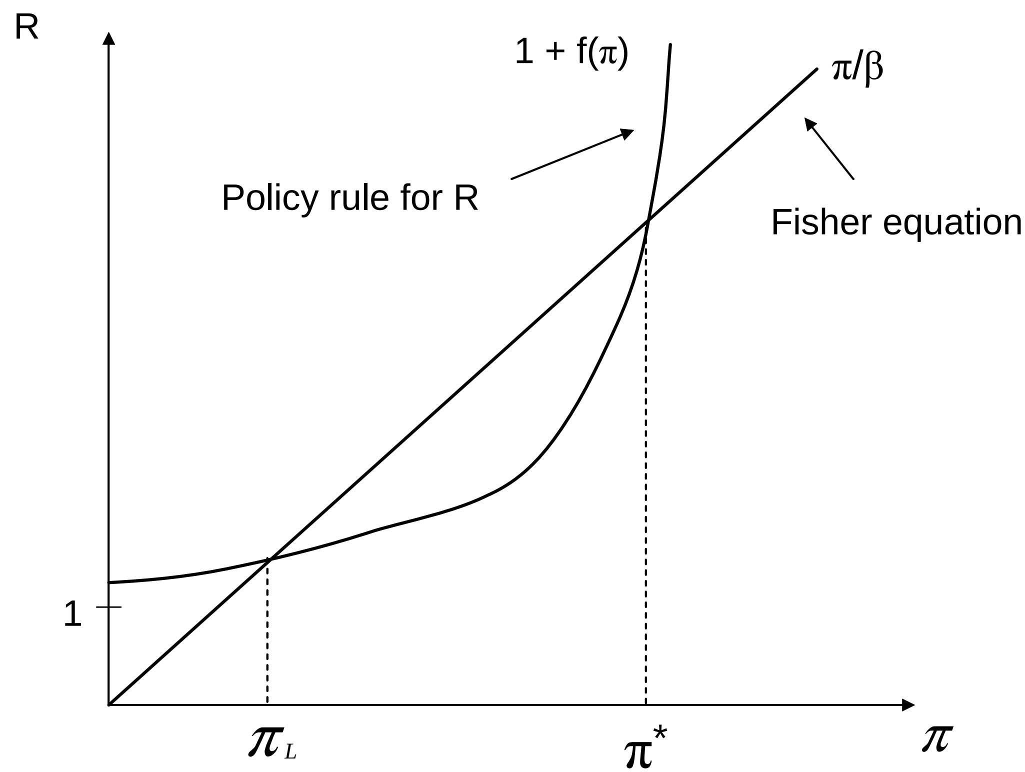
Evans, Guse & Honkapohja (EER, 2008), Evans & Honkapohja (2009)

Background

- A standard characterization of **monetary policy** is that the CB (Central Bank) follows a Taylor-type rule in which the **interest rate** R_t **responds more than one-for-one to inflation** π_t near the inflation target π^*.

- The ZLB (**zero lower bound**) for $R_t - 1$ **implies** a **second unintended steady-state** π_L of any (continuous) "global" Taylor rule. See Figure.

- Under perfect foresight (& rational expectations) the low steady state is "indeterminate," i.e. has multiple perfect foresight paths that converge to it (a low R "**liquidity trap**"). See Benhabib, Schmitt-Grohe and Uribe (2001, 2002).



Multiple steady states with global Taylor rule.

Here R = nominal interest rate factor (e.g. 1.06), π = inflation factor (e.g. 1.02), and β^{-1} = steady state real interest rate factor (e.g. β = 0.96).

Outline

- We consider a fairly standard NK (New Keynesian) model with a **global Taylor rule** and a **standard fiscal policy** setting.

- We consider the solutions **both** under **RE** (rational expectations) **and** when private agents form expectations of future consumption and inflation using **adaptive learning**.

- We find: **under learning** the π^* **solution** is **locally stable**, but if expectations are too pessimistic they follow unstable paths – **deflationary spirals**.

- To prevent deflationary spirals we consider procedures that suspend normal policies and replace them with **aggressive policies when π falls to some threshold $\tilde{\pi} < \pi^*$.**

- **Fiscal as well** as aggressive **monetary** policy may be needed.

- The aggressive policies must be based on an **inflation threshold**. Using an **output** threshold is **not sufficient**.

The Model

We use a standard discrete-time, stochastic NK ("New Keynesian") model.

A continuum of households produce a differentiated consumption good under conditions of
(i) monopolistic competition and
(ii) price-adjustment costs.

Private Sector

Households maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U_{t,j} \left(c_{t,j}, \frac{M_{t-1,j}}{P_t}, h_{t,j}, \frac{P_{t,j}}{P_{t-1,j}} - 1 \right)$$

$$st.\ c_{t,j} + m_{t,j} + b_{t,j} + \Upsilon_{t,j} = m_{t-1,j}\pi_t^{-1} + R_{t-1}\pi_t^{-1} b_{t-1,j} + \frac{P_{t,j}}{P_t} y_{t,j},$$

where $c_{t,j}$ is the consumption aggregator of agent j, where

$$y_{t,j} = h_{t,j}^{\alpha},\ P_{t,j} = \left(\frac{y_{t,j}}{Y_t} \right)^{-1/\nu} P_t$$

and

$$U_{t,j} = \frac{c_{t,j}^{1-\sigma_1}}{1-\sigma_1} + \frac{\chi}{1-\sigma_2} \left(\frac{M_{t-1,j}}{P_t} \right)^{1-\sigma_2} - \frac{h_{t,j}^{1+\varepsilon}}{1+\varepsilon} - \frac{\gamma}{2} \left(\frac{P_{t,j}}{P_{t-1,j}} - 1 \right)^2.$$

Monetary and Fiscal Policy

Government purchases g_t are given by

$$g_t = \bar{g} + u_t$$

where u_t is exogenous AR(1), stationary. Lump-sum taxes Υ_t are given by

$$\Upsilon_t = \kappa_0 + \kappa b_{t-1} + \eta_t,$$

η_t white noise, and $\beta^{-1} - 1 < \kappa < 1$. Real 1-period debt b_t evolves as:

$$b_t + m_t + \Upsilon_t = g_t + m_{t-1}\pi_t^{-1} + R_{t-1}\pi_t^{-1}b_{t-1}.$$

Monetary Policy:

$$R_t - 1 = \theta_t f\left(\pi_t\right), \text{ where } \theta_t \text{ is AR(1) with } E\theta_t = 1$$

and $f(\pi)$ as shown earlier $\longrightarrow$ two steady states $0 < \pi_L < \pi^*$.

Key Equations

The equilibrium of the model is given by the monetary and fiscal policy, market clearing, and the private-sector optimization equations:

$$\frac{\alpha\gamma}{\nu}(\pi_t - 1)\pi_t = h_t\left(h_t^{\varepsilon} - \alpha\left(1 - \frac{1}{\nu}\right)h_t^{\alpha-1}c_t^{-\sigma_1}\right)$$
$$+\beta\frac{\alpha\gamma}{\nu}E_t\left[(\pi_{t+1} - 1)\pi_{t+1}\right],$$

$$c_t^{-\sigma_1} = \beta R_t E_t\left(\pi_{t+1}^{-1}c_{t+1}^{-\sigma_1}\right),$$

and a money demand equation.

The first equation is a **NK Phillips curve**. The second is the Euler equation for c_t (the **NK IS curve**).

Rational Expectations

Consider the stochastic system for "**small shocks**," i.e. if the random exogenous shocks have small support, and RE.

For the (c_t, π_t) block there is a **stochastic steady-state** solution

$$\begin{pmatrix} c_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} c \\ \pi \end{pmatrix} + \begin{pmatrix} G_{cu} & G_{c\theta} \\ G_{\pi u} & G_{\pi\theta} \end{pmatrix} \begin{pmatrix} u_t \\ \tilde{\theta}_t \end{pmatrix},$$

near each of the two steady states.

Proposition 1. *In the linearized model there are two steady states $\pi^* > \pi_L$. For $\gamma > 0$ sufficiently small, the steady state $\pi = \pi^*$ is locally determinate (i.e. locally unique) and the steady state $\pi = \pi_L$ is locally indeterminate.*

We now consider the situation under adaptive learning.

Learning and Expectational Stability

We now replace RE by **private agent learning**.

We **return to the nonlinear system** of equations so that we can study the **global properties** of the system **under learning**.

We assume that agents estimate the linear projection

$$\begin{aligned} c_{t+1} &= a_c + du_t + e\theta_t + \varepsilon_{c,t+1} \\ \pi_{t+1} &= a_\pi + fu_t + g\theta_t + \varepsilon_{\pi,t+1}, \end{aligned}$$

and use it to make forecasts.

Timing: At end of period $t-1$, agents use LS to update estimated coefficients $a_{c,t-1}, d_{t-1}, e_{t-1}, a_{\pi,t-1}, f_{t-1}, g_{t-1}$. Then, at the start of t agents form forecasts

$$\begin{aligned} c_{t+1}^e &= a_{c,t-1} + d_{t-1}u_t + e_{t-1}\theta_t \\ \pi_{t+1}^e &= a_{\pi,t-1} + f_{t-1}u_t + g_{t-1}\theta_t. \end{aligned}$$

This determines actual c_t, π_t. Then at the end of t the coefficients are updated using the new data point.

Do estimates and forecasts converge (approximately) to RE corresponding to the π^* or π_L equilibrium? This can be analyzed using **E-stability**.

We make a **simplification** that does not affect any of our key results. It turns out that stability is governed by the stability of the intercepts, not the coefficients for exogenous shocks.

Thus **for simplicity** we now **assume that** u_t **and** θ_t **are** iid and drop them from the regression.

In effect, private **agents simply estimate the unknown means** of π_t and c_t. Coefficient updating is then given by

$$\begin{aligned} \pi^e_{t+1} &= \pi^e_t + \phi_t(\pi_{t-1} - \pi^e_t) \\ c^e_{t+1} &= c^e_t + \phi_t(c_{t-1} - c^e_t), \end{aligned}$$

where ϕ_t is the "**gain sequence**" (e.g. $\phi_t = t^{-1}$, i.e. decreasing gain, or $\phi_t = \phi$ for $0 < \phi < 1$, i.e. constant gain).

The **system under learning** consists of the original system but with RE replaced by adaptive learning.

The π_t, c_t block under learning is given by

$$\frac{\alpha\gamma}{\nu}(\pi_t - 1)\,\pi_t = \beta\frac{\alpha\gamma}{\nu}\left(\pi^e_{t+1} - 1\right)\pi^e_{t+1} + (c_t + g_t)^{(1+\varepsilon)/\alpha}$$
$$-\alpha\left(1 - \frac{1}{\nu}\right)(c_t + g_t)c_t^{-\sigma_1}$$

$$c_t = c^e_{t+1}(\pi^e_{t+1}/\beta R_t)^{\sigma_1},$$

together with the interest-rate rule for R_t. c^e_{t+1}, π^e_{t+1} are given by adaptive learning, as above. Note: we are assuming **Euler-equation learning**.

These **"temporary equilibrium" PC and IS equations** determine π_t, c_t given expectations. The **temporary equilibrium system** is completed by the money equation and the bond evolution equation.

Under learning does the system evolve towards π^* or towards π_L?

Stability under learning

Formally we can write the temporary equilibrium system as

$$\begin{aligned}\pi_t &= F_\pi(\pi^e_{t+1}, c^e_{t+1}, u_t, \theta_t) \\ c_t &= F_c(\pi^e_{t+1}, c^e_{t+1}, u_t, \theta_t),\end{aligned}$$

with

$$\begin{aligned}\pi^e_{t+1} &= \pi^e_t + \phi_t(\pi_{t-1} - \pi^e_t) \\ c^e_{t+1} &= c^e_t + \phi_t(c_{t-1} - c^e_t).\end{aligned}$$

This is a stochastic recursive algorithm, whose convergence properties can be analyzed, as usual, using E-stability.

Local stability under learning is determined by E-stability. A stochastic steady state is **E-stable** if the differential equation

$$\begin{pmatrix} d\pi^e/d\tau \\ dc^e/d\tau \end{pmatrix} = \begin{pmatrix} T_\pi(\pi^e, c^e) \\ T_c(\pi^e, c^e) \end{pmatrix} - \begin{pmatrix} \pi^e \\ c^e \end{pmatrix}$$

is locally asymptotically stable at the steady state (π, c), where

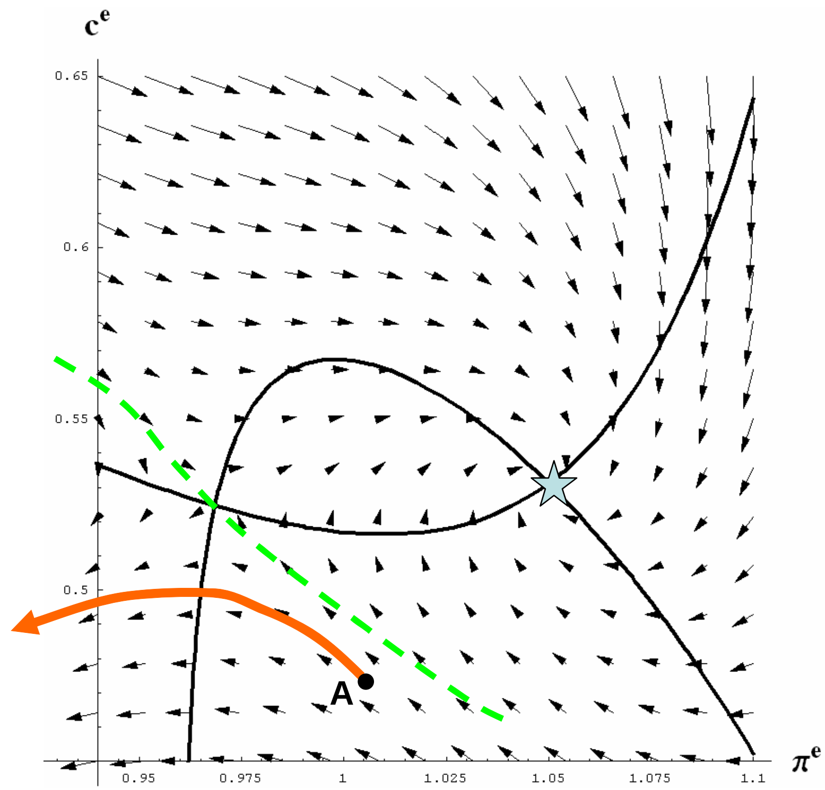
$$\begin{aligned} T_\pi(\pi^e, c^e) &= EF_\pi(\pi^e, c^e, u_t, \theta_t) \\ T_c(\pi^e, c^e) &= EF_c(\pi^e, c^e, u_t, \theta_t). \end{aligned}$$

$T(\pi^e, c^e)$ maps the **Perceived Law of Motion** to the **Actual Law of Motion**.

Proposition 2. *For $\gamma > 0$ sufficiently small, the (stochastic) steady state at $\pi = \pi^*$ is locally stable under learning and the steady state at $\pi = \pi_L$ is locally unstable under learning, taking the form of a saddle point.*

See Figure. **Local stability of $\boldsymbol{\pi}^*$** is reassuring, **but** the instability under learning of π_L comes with a **danger**: the possibility of **deflationary spirals leading to stagnation** (a **deflation trap**).

If expectations π^e, c^e are initially low enough then actual π, c, and output y are low and this is self-reinforcing under learning.



π^e and c^e dynamics under normal policy

Adding Aggressive Monetary Policy

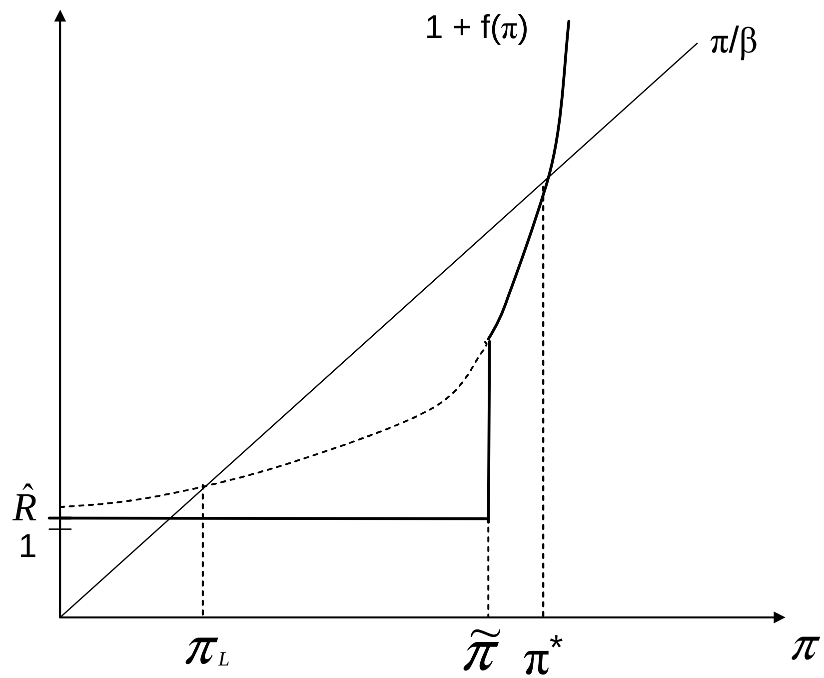
Can the deflation trap be avoided if modify monetary policy to be more aggressive when we approach the expectational danger zone? We consider the following **change to monetary policy**:

$$R_t = \begin{cases} 1 + \theta_t f(\pi_t) \text{ if } \pi_t > \tilde{\pi} \\ \hat{R} \text{ if } \pi_t < \tilde{\pi}, \end{cases}$$

where $\hat{R} > 1$ is close to the ZLB of 1, and

$$\hat{R} \leq R_t \leq 1 + \theta_t f(\pi_t) \text{ if } \pi_t = \tilde{\pi}.$$

Thus if π_t threatens to fall below some **threshold** $\tilde{\pi}$, we suspend the global Taylor rule and reduce R_t as needed to try to maintain $\pi_t = \tilde{\pi}$, if necessary reducing R_t all the way to $\hat{R}$. See Figure.

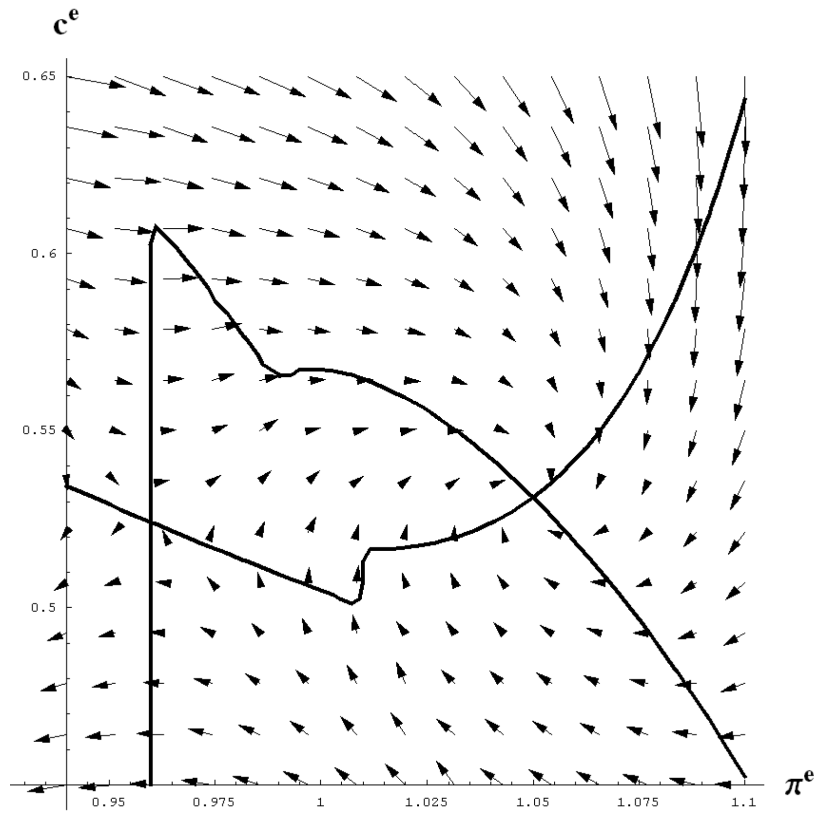


Aggressive monetary policy for $\pi \leq \tilde{\pi}$.

It turns out that **aggressive monetary policy is not enough** to avoid deflationary spirals.

Proposition 3. *There is a steady state at* $\hat{\pi} = \beta \hat{R}$ *and there is no steady state value for* π_t *below* $\hat{\pi}$*. For all* $\gamma > 0$ *sufficiently small the steady state at* $\hat{\pi} = \beta \hat{R}$ *is a saddle point under learning.*

While the region of stability may increase, the possibility of a deflationary trap remains.



Two steady states with standard fiscal policy and $\pi_L < \tilde{\pi} < \pi^*$.

Combined Monetary and Fiscal Policy

Our **recommended policy** is to add an inflation floor or **threshold** $\tilde{\pi}$, with $\pi_L < \tilde{\pi} < \pi^*$, and to use **both** aggressive monetary and fiscal policy, if needed, to **ensure this floor is achieved**. If $\pi_L < 1$ then $\tilde{\pi} = 1$ can be used.

Our policy is feasible because fiscal policy can guarantee an inflation floor:

Lemma *Given expectations* c^e_{t+1} *and* π^e_{t+1} *and setting* $R_t = \hat{R}$, *any value of* $\pi_t > 1/2$ *can be achieved by setting* g_t *sufficiently high.*

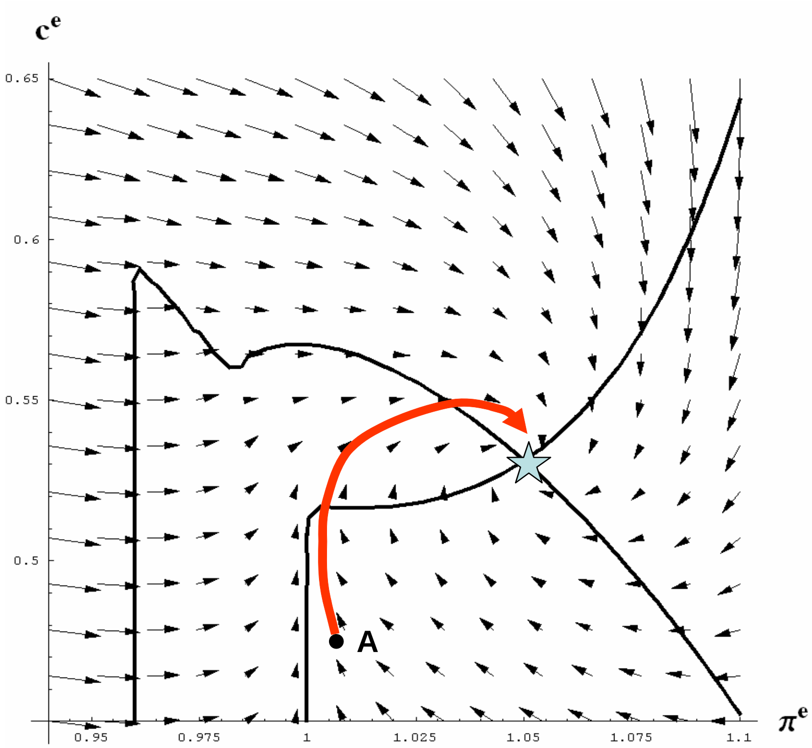
This follows by implicitly differentiating the Phillips curve equation.

Our recommended policy: Follow normal monetary and fiscal policy provided $\pi_t \geq \tilde{\pi}$. Reduce R_t as needed and increase g_t if necessary to ensure $\pi_t \geq \tilde{\pi}$.

Policy needs to focus on inflation, not expansionary spending *per se*.

Proposition 4. *If $\pi_L < \tilde{\pi} < \pi^*$ then π^* is the unique steady state and it is stable under learning.*

Thus for $\pi_L < \tilde{\pi} < \pi^*$ **our** recommended **policy eliminates the deflation trap**. See Figure.



Inflation threshold $\tilde{\pi}$, $\pi_L < \tilde{\pi} < \pi^*$, for aggressive monetary policy and, if needed, aggressive fiscal policy.

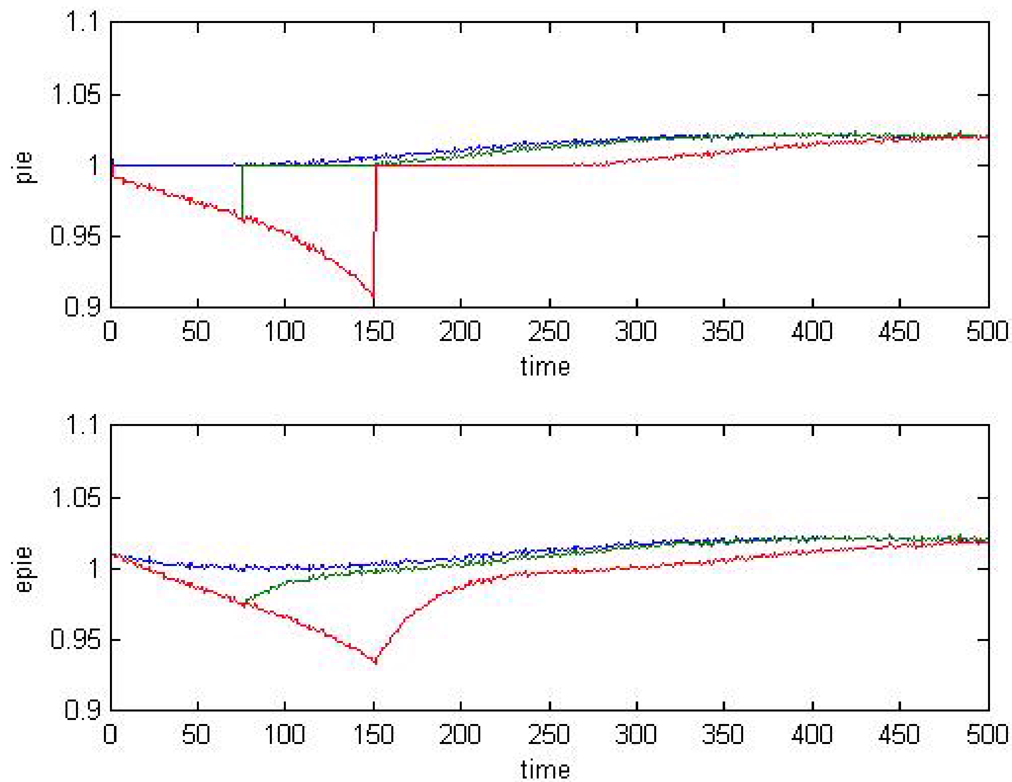
Stochastic Simulations

We set $\pi^* = 1.02$ (with $\pi_L = 0.975$), $\tilde{\pi} = 1$ and $\phi = 1/30$. We **start with a pessimistic expectations shock** at $t = 0$ (π^e falls to 1.01 and c^e falls by about 8%), large enough to lead to deflationary spirals.

We examine the paths if initially normal policies are used, and then **our recommended policy is introduced at** $t_1 = 150$ vs. $t_1 = 80$. These are compared to the results if the policy is initially in place. See Figs.

Introducing our policy earlier, at $t_1 = 80$ **avoids the worst** part of the stagnation. **Having the policy in place** when the shock occurs **is best**.

Setting π^* higher, e.g. $\pi^* = 1.05$ can avoid the need for fiscal policy for the larger shock. However there is an efficiency loss of a higher inflation target.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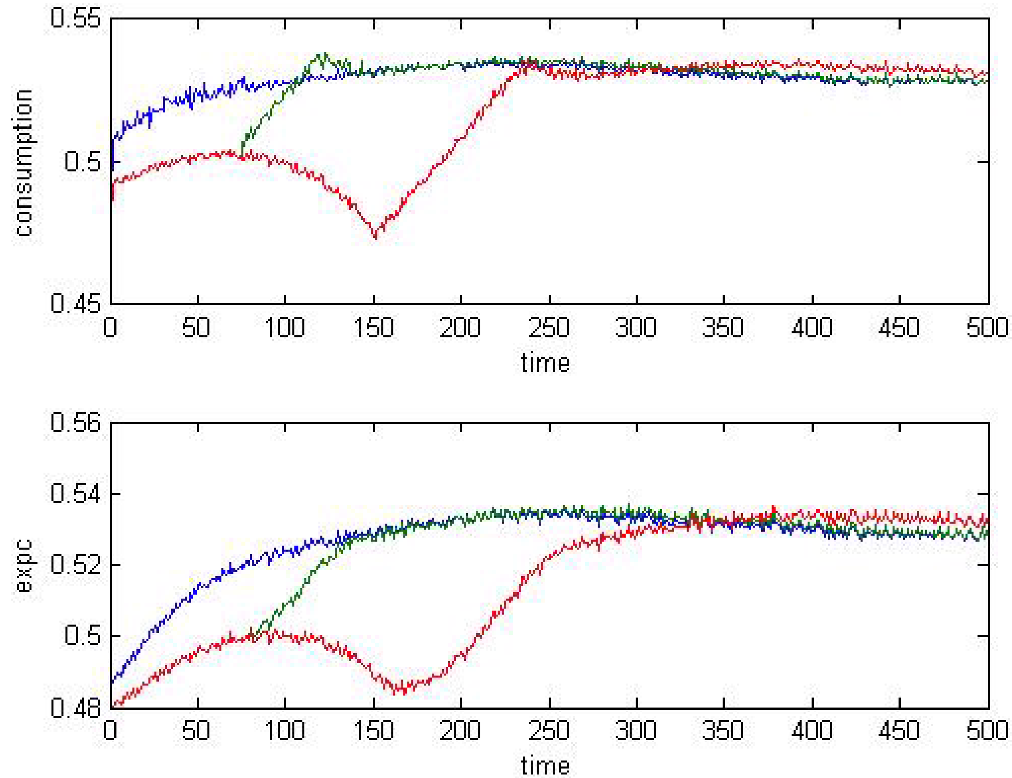

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Extension: infinite horizon learning

– Our preceding analysis was under the assumption that agents' decision rules had a short planning horizon, based on subjective Euler equations.

– Commitment to low interest rates cannot be studied in that setting.

– In Evans & Honkapohja (2009) we consider a modification of the set-up. We replace Euler-equation learning with infinite-horizon decision rules, as in Marcet and Sargent 1989, Preston 2005, 2006 or Evans, Honkapohja & Mitra (2009, 2010).

– In this setting agents solve forward their Euler equations & use their life-time budget constraint. Now under learning they must, at each time, forecast the whole future time path. The temporary equilibrium equations are

$$
\begin{aligned}
Q_t &= \frac{\nu}{\gamma}\sum_{j=0}^{\infty}\alpha^{-1}\beta^{j}\left(y^e_{t+j}\right)^{(1+\varepsilon)/\alpha}-\frac{\nu-1}{\gamma}\sum_{j=0}^{\infty}\beta^{j}\left(\frac{y^e_{t+j}}{x^e_{t+j}}\right), \\
\text{where } Q_t &= (\pi_t-1)\,\pi_t, \text{ and } x^e_{t+j}=y^e_{t+j}-g^e_{t+j}, \text{ and} \\
c_t &= (1-\beta)\left(y_t-g_t+\sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}x^e_{t+j}\right).
\end{aligned}
$$

– Our central results extend to this setting. We continue to find that fiscal policy may be needed. Monetary policy alone, even if policy commits to zero net interest rates for ever, may be insufficient to avoid the deflation trap.

Conclusions

- **We take seriously the multiple equilibrium problem** emphasized by the RE literature on the ZLB.

- However, **the adaptive learning approach provides another perspective and is** in some ways **more alarming**: large pessimistic shocks can lead to unstable deflationary spirals.

- To avoid this **normal policy must be replaced by aggressive monetary and fiscal policy** triggered if inflation falls below a **threshold** $\tilde{\pi} > \pi_L$.

- Output thresholds are inadequate. **The key is to stabilize inflation**.

Conclusions to Lectures

- Expectations play a large role in modern macroeconomics. People are smart, but boundedly rational. Cognitive consistency principle: economic agents should be about as smart as (good) economists, e.g. model agents as econometricians.

- Stability of RE under private agent learning is not automatic. Monetary policy must be designed to ensure both determinacy and stability under learning.

- Policymakers may need to use policy to guide expectations. Under learning there is the possibility of persistent deviations from RE, hyperinflation, and deflationary spirals with stagnation. Appropriate monetary and fiscal policy design can minimize these risks.