Adaptive Learning

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J. C. Trichet: "Understanding expectations formation as a process underscores the strategic interdependence that exists between expectations formation and economics." (Zolotas lecture, 2005)

Ben S. Bernanke: "In sum, many of the most interesting issues in contemporary monetary theory require an analytical framework that involves learning by private agents and possibly the central bank as well." (NBER, July 2007).

Outline

Introduction
— Muth/Lucas model with LS learning
— The E-stability principle
— Implications of learning for theory and policy, Methodological issues, Learning and empirical research

Application to the New Keynesian Model
— Structure & monetary policy rules
— Determinacy, LS learning and E-stability
— Results for different interest rate rules

Four other applications: (i) Monetary Policy under Perpetual Learning, (ii) Explaining Hyperinflations, (iii) Bubbles and Crashes, (iv) Liquidity Traps

Conclusions

Introduction

– Macroeconomic models are usually based on optimizing agents in dynamic, stochastic setting and can be summarized by a **dynamic system,** e.g.

$$\begin{aligned} y_t &= Q(y_{t-1}, y^e_{t+1}, w_t) \\ \text{or } y_t &= Q(y_{t-1}, \left\{y^e_{t+1}\right\}_{j=0}^{\infty}, w_t) \end{aligned}$$

$y_t =$ vector of economic variables at time t (unemployment, inflation, investment, etc.), $y^e_{t+1} =$ expectations of these variables, $w_t =$ exogenous random factors at t.

– The presence of **expectations** y^e_{t+1} makes macroeconomics inherently different from natural science. But **how are expectations formed?**

– Since Lucas (1972, 1976) and Sargent (1973) the standard assumption is **rational expectations** (RE).

– RE assumes too much knowledge & coordination for economic agents. We need a **realistic** model of **rationality** What form should this take?

– My general answer is given by the **Cognitive Consistency Principle**: economic agents should be about as smart as (good) economists. This still leaves open various possibilities, e.g.

- model agents like **economic theorists** – the **eductive** approach, or

- model them like **econometricians** – the **adaptive** (or evolutive) approach.

– In this talk I follow the adaptive approach. Agent/econometricians must select models, estimate parameters and update their models over time.

A Muth/Lucas-type Model

Consider a simple univariate reduced form:

$$p_t = \mu + \alpha E^*_{t-1} p_t + \delta' w_{t-1} + \eta_t. \qquad \text{(RF)}$$

$E^*_{t-1} p_t$ denotes expectations of p_t formed at $t-1$, w_{t-1} is a vector of exogenous observables and η_t is an unobserved iid shock.

Muth cobweb example. Demand and supply equations:

$$\begin{aligned} d_t &= m_I - m_p p_t + v_{1t} \\ s_t &= r_I + r_p E^*_{t-1} p_t + r'_w w_{t-1} + v_{2t}, \end{aligned}$$

$s_t = d_t$, yields (RF) where $\alpha = -r_p/m_p < 0$ if $r_p, m_p > 0$.

Lucas-type monetary model. AS + AD + monetary feedback:

$$\begin{aligned} q_t &= \bar{q} + \pi(p_t - E^*_{t-1} p_t) + \zeta_t, \\ m_t + v_t &= p_t + q_t \text{ and } m_t = \bar{m} + u_t + \rho' w_{t-1} \end{aligned}$$

leads to yields (RF) with $0 < \alpha = \pi/(1+\pi) < 1$.

Rational Expectations vs. Least-Squares Learning

The model $p_t = \mu + \alpha E_{t-1} p_t + \delta' w_{t-1} + \eta_t$. has the **unique REE**

$$\begin{aligned} p_t &= \bar{a} + \bar{b}' w_{t-1} + \eta_t, \text{ where} \\ \bar{a} &= (1-\alpha)^{-1}\delta \text{ and } \bar{b} = (1-\alpha)^{-1}\delta. \end{aligned}$$

Under **LS learning**, agents have the beliefs or perceived law of motion (PLM)

$$p_t = a + b w_{t-1} + \eta_t,$$

but a, b are unknown. At the end of time $t-1$ they estimate a, b by LS (Least Squares) using data through $t-1$. Then they use the estimated coefficients to make forecasts $E^*_{t-1} p_t$.

– End of $t-1$: w_{t-1} and p_{t-1} observed. Agents update estimates of a, b to a_{t-1}, b_{t-1} using $\{p_s, w_{s-1}\}_{s=1}^{t-1}$. Agents make forecasts

$$E^*_{t-1} p_t = a_{t-1} + b'_{t-1} w_{t-1}.$$

– **Temporary equilibrium at t:** (i) p_t is determined as

$$p_t = \mu + \alpha E^*_{t-1} p_t + \delta' w_{t-1} + \eta_t$$

and w_t is realized. (ii) agents update estimates to a_t, b_t and forecast

$$E^*_t p_{t+1} = a_t + b'_t w_t.$$

The fully specified dynamic system under LS learning is written recursively as

$$\begin{aligned}
E^*_{t-1} p_t &= \phi'_{t-1} z_{t-1} \text{ where } \phi'_{t-1} = (a_{t-1}, b'_{t-1}) \text{ and } z'_{t-1} = (1, w_{t-1}) \\
p_t &= \mu + \alpha E^*_{t-1} p_t + \delta' w_{t-1} + \eta_t, \\
\phi_t &= \phi_{t-1} + t^{-1} R_t^{-1} z_{t-1} (p_t - \phi'_{t-1} z_{t-1}) \\
R_t &= R_{t-1} + t^{-1} (z_{t-1} z'_{t-1} - R_{t-1}),
\end{aligned}$$

Question: Will $(a_t, b_t) \to (\bar{a}, \bar{b})$ as $t \to \infty$?

Theorem (Bray & Savin (1986), Marcet & Sargent (1989)). Convergence to RE, i.e. $(a_t, b_t') \rightarrow (\bar{a}, \bar{b}')$ a.s. if $\alpha < 1$. If $\alpha > 1$ convergence with prob. 0.

Thus the REE is stable under LS learning both for Muth model ($\alpha < 0$) and Lucas model ($0 < \alpha < 1$), but is not stable if $\alpha > 1$. The stability condition can be obtained using the **E-stability principle** based on an associated ODE.

Instability arises for $\alpha > 1$ because economy under learning is **self-referential**.

For a wide range of models **E-stability** has been shown to govern stability under LS learning, see Evans & Honkapohja (1992, 2001, etc.).

E-STABILITY

Proving the theorem relies on stochastic approximation theorems. However, there is an easy way of deriving the stability condition $\alpha < 1$ that is quite general. Start with the PLM

$$p_t = a + b' w_{t-1} + \eta_t,$$

and consider what would happen if (a, b) were fixed at some value possibly different from the RE values $(\bar{a}, \bar{b})$. The corresponding expectations are

$$E^*_{t-1} p_t = a + b' w_{t-1},$$

which would lead to the Actual Law of Motion (ALM)

$$p_t = \mu + \alpha(a + b' w_{t-1}) + \delta' w_{t-1} + \eta_t.$$

The implied ALM gives the mapping T: PLM $\longrightarrow$ ALM:

$$T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \mu + \alpha a \\ \delta + \alpha b \end{pmatrix}.$$

The REE $\bar{a}, \bar{b}$ is a fixed point of T. Expectational-stability (“E-stability) is defined by the differential equation

$$\frac{d}{d\tau}\begin{pmatrix} a \\ b \end{pmatrix} = T\begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} a \\ b \end{pmatrix}.$$

Here τ denotes artificial or notional time. $\bar{a}, \bar{b}$ is said to be E-stable if it is stable under this differential equation.

In the current case the T-map is linear. Component by component we have

$$\frac{da}{d\tau} = \mu + (\alpha - 1)a \text{ and } \frac{db_i}{d\tau} = \delta + (\alpha - 1)b_i \text{ for } i = 1, ..., p.$$

It follows that the REE is E-stable if and only if $\alpha < 1$. This is the stability condition, given in the theorem, for stability under LS learning.

Intuition: under LS learning the parameters a_t, b_t are slowly adjusted, on average, in the direction of the corresponding ALM parameters.

For **discounted LS** the "gain" t^{-1} is replaced by a (typically small) constant $0 < \gamma < 1$, e.g. $\gamma = 0.04$. Often called "constant gain" learning

With constant gain recursive LS and $\alpha < 1$ convergence is to a stochastic process near $(\bar{a}, \bar{b})$.

The E-Stability Principle

– The E-stability technique works quite generally.

– To study convergence of LS learning to an REE, specify a PLM with parameters ϕ. The PLM can be thought of as an econometric forecasting model. The REE is the PLM with $\phi = \bar{\phi}$.

– PLMs can take the form of ARMA or VARs or admit cycles or a dependence on sunspots.

– Compute the ALM for this PLM. This gives a map

$$\phi \rightarrow T(\phi),$$

with fixed point $\bar{\phi}$.

– E-stability is determined by local asymptotic stability of $\bar{\phi}$ under

$$\frac{d\phi}{d\tau} = T(\phi) - \phi.$$

The E-stability condition: eigenvalues of $DT(\bar{\phi})$ have real parts less than 1.

– The E-stability principle: E-stability governs local stability of an REE under LS and closely related learning rules.

– E-stability can be used as a selection criterion in models with multiple REE.

– The techniques can be applied to multivariate linearized models, and thus to RBC, OLG, New Keynesian and DSGE models.

– Iterative E-stability, $\lim_{n\to\infty} T^n(\phi) = \bar{\phi}$, plays a role in eductive learning.

Multiple Equilibria

Adaptive learning can be applied to models with multiple REE.

- Multiple steady states in nonlinear models, e.g. OG or endog growth models with seigniorage, increasing returns or externalities, e.g. Howitt&McAfee, Evans,Honkapohja&Romer.

- Cycles and sunspot equilibria in forward-looking nonlinear models, e.g. Guesnerie&Woodford, Woodford, EH, EH&Marimon.

- Sunspot equilibria in linearized models with indeterminate steady states.

General Implications of Learning Theory

- Can assess **plausibility** of RE based on stability under LS learning

- Use local stability under learning as a **selection criterion** in models with multiple REE

- **Persistent learning dynamics** that arise with modified learning rules that allow for:

  (i) discounting older data to allow for possible structural shifts.

  (ii) model selection when the specification is uncertain

- **Policy implications**: Policy should facilitate learning by private agents of the targeted REE.

Methodological Issues

- **Misspecification**. Like applied econometricians, agents may use misspecified models $\longrightarrow$ restricted perceptions equilibria (EH, Sargent, E&Ramey)).

- **Discounted LS & structural change.** Agents may be concerned about structural change and discount older data $\longrightarrow$ escape dynamics. (Sargent, N. Williams)

- **Heterogeneous expectations.** Can introduce through heterogeneity in priors, econometric learning rules, inertia, forecasting models, etc. (Bay&Savin, EH&Marimon, HMitra)

- **Multiple forecasting models.** Dynamic predictor selection (Brock&Hommes, Branch&Evans) or Bayesian model averaging (Cogley&Sargent).

- **Planning horizon.** Infinitely-lived agents can engage in short-horizon decision making (Euler-equation learning, EH,E&McGough), or using infinite-horizon learning (Bruce Preston).

- **Extent of structural knowledge.** Partial structural knowledge can be combined with adaptive learning. (EH&Mitra)

- **Precise information set.** Stability may depend, e.g., on whether aggregate endogenous variables are observed at t.

Learning and Empirical Research

- Inflation: (i) Rise and fall of inflation (Sargent 1999, Primaceri 2006, Orphanides & Williams 2005a,c

  (ii) Latin American inflation (Marcet and Nicolini 2003)

- Real business cycle applications (Williams 2004, Giannitsarou 2006, Eusepi and Preston forthcoming AER)

- Asset prices and learning (Timmermann 1993,1996, Brock & Hommes 1998, Chakraborty & Evans 2008, Lansing 2010, Branch & Evans forthcoming, Adam, Marcet & Nicolini)

- Estimated NK models with learning (Milani, 2007, forthcoming EJ).

The New Keynesian (NK) Model

- Log-linearized New Keynesian model (Clarida, Gali and Gertler 1999 and Woodford 2003 etc.). NK "IS" and "Phillips" curves

$$x_t = -\varphi(i_t - E_t^* \pi_{t+1}) + E_t^* x_{t+1} + g_t$$

$$\pi_t = \lambda x_t + \beta E_t^* \pi_{t+1} + u_t,$$

  where x_t =output gap, π_t =inflation, i_t = nominal interest rate. $\varphi, \lambda > 0$ and $0 < \beta < 1$. Observable shocks g_t, u_t are stationary AR(1).

- Many versions of the NK model incorporate **inertia**, i.e. π_{t-1} or x_{t-1}.

- Assumes "Euler-equation learning". Learning with IH decisions has also been examined (Preston).

Policy rules for the interest rate i_t

- Standard **Taylor rule**, e.g.

$$\begin{aligned} i_t &= \chi_\pi \pi_t + \chi_x x_t \text{ where } \chi_\pi, \chi_x > 0, \text{ or} \\ i_t &= \chi_\pi E_t^* \pi_{t+1} + \chi_x E_t^* x_{t+1} \end{aligned}$$

  For determinacy & learning stability see Bullard & Mitra (JME, 2002).

- **Optimal monetary policy**: Under commitment minimize loss

$$E_t \sum_{s=0}^{\infty} \beta^s \left[\pi_{t+s}^2 + \alpha x_{t+s}^2 \right].$$

  We get the (timeless perspective) **optimal "targeting rule"** (Woodford, various)

$$\lambda \pi_t + \alpha (x_t - x_{t-1}) = 0.$$

- One can attempt to implement optimal policy by various i_t rules:

1. "Fundamentals-based" reaction function

$$i_t = \psi_x x_{t-1} + \psi_g g_t + \psi_u u_t$$

with coefficients obtained from the RE solution under optimal policy.

2. Expectations-based reaction function

$$i_t = \delta_L x_{t-1} + \delta_\pi E_t^* \pi_{t+1} + \delta_x E_t^* x_{t+1} + \delta_g g_t + \delta_u u_t$$

with coefficients obtained from IS, PC & optimal targeting rule, e.g. $\delta_\pi = 1 + \lambda\beta/(\varphi(\alpha + \lambda^2))^{-1}$.

3. Various hybrid rules have also been proposed.

Determinacy and Stability under Learning

DETERMINACY

Combining IS, PC and an i_t rule leads to a bivariate reduced form in x_t and π_t.Letting $y_t' = (x_t, \pi_t)'$ and $v_t' = (g_t, u_t)'$ the model can be written

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = M \begin{pmatrix} E_t^* x_{t+1} \\ E_t^* \pi_{t+1} \end{pmatrix} + N \begin{pmatrix} x_{t-1} \\ \pi_{t-1} \end{pmatrix} + P \begin{pmatrix} g_t \\ u_t \end{pmatrix},$$

$$y_t = M E_t^* y_{t+1} + N y_{t-1} + P v_t.$$

If the model is determinate there is a unique stationary REE, taking the form

$$y_t = \bar{b} y_{t-1} + \bar{c} v_t.$$

Determinacy condition: compare # of stable eigenvalues of matrix of stacked first-order system to # of predetermined variables. If "indeterminate" there are multiple solutions, which include stationary sunspot solutions.

LEARNING

Under LS learning, suppose agents have a "minimal state variable" PLM

$$y_t = a + by_{t-1} + cv_t,$$

where we now allow for an intercept, and estimate (a_t, b_t, c_t) in period t based on past data.

- Forecasts are computed from the estimated PLM.
- New data is generated according to the model with the given forecasts.
- Estimates are updated to $(a_{t+1}, b_{t+1}, c_{t+1})$ using least squares.
- Convergence $(a_t, b_t, c_t) \rightarrow (0, \bar{b}, \bar{c})$ is governed by E-stability.

E-STABILITY METHODOLOGY

Linear economic model

$$y_t = ME_t^* y_{t+1} + N y_{t-1} + P v_t.$$

Under the PLM (Perceived Law of Motion)

$$y_t = a + b y_{t-1} + c v_t.$$

$$E_t^* y_{t+1} = (I + b)a + b^2 y_{t-1} + (bc + cF) v_t.$$

This $\longrightarrow$ ALM (Actual Law of Motion)

$$y_t = M(I + b)a + (Mb^2 + N) y_{t-1} + (Mbc + NcF + P) v_t.$$

This gives a **mapping from PLM to ALM:**

$$T(a,b,c) = (M(I+b)a, Mb^2 + N, Mbc + NcF + P).$$

The optimal REE is a fixed point of $T(a,b,c)$. If

$$d/d\tau(a,b,c) = T(a,b,c) - (a,b,c)$$

is locally asymptotically stable at the REE it is said to be **E-stable**. The **E-stability conditions** can be stated in terms of the derivative matrices

$$\begin{aligned} DT_a &= M(I+\bar{b}) \\ DT_b &= \bar{b}' \otimes M + I \otimes M\bar{b} \\ DT_c &= F' \otimes M + I \otimes M\bar{b}, \end{aligned}$$

where $\otimes$ denotes the Kronecker product and $\bar{b}$ denotes the REE value of b.

E-stability governs stability under LS learning.

Back to NK model: Bullard & Mitra show determinacy & E-stability if $i_t = \chi_\pi \pi_t + \chi_x x_t$ with $\chi_\pi > 1$, $\chi_x > 0$.

But policymakers seem to use $i_t = \chi_\pi E_t^* \pi_{t+1} + \chi_x E_t^* x_{t+1}$, which can in some cases lead to indeterminacy (Bernanke & Woodford).

Stationary sunspot equilibria (SSE). Can indeterminacy $\longrightarrow$ SSEs that are stable under learning? This has been established in a variety of nonlinear & linear models, e.g.: OG model of money (Woodford, 1990), Animal Spirits (Howitt & McAfee, 1992), Growth Cycles (Evans, Honkapohja and Romer, 1998), Cash-in-Advance seigniorage models (Evans, Honkapohja and Marimon, 2007), Hyperinflation models (Adam, Evans and Honkapohja, 2003).

Can SSEs in New Keynesian Model be stable under learning? Honkapohja and Mitra (JME, 2004) and Evans and McGough (JEDC, 2005ab) find:

1. In many cases with indeterminacy, SSEs are not stable under learning. For example, if $i_t = \chi_\pi \pi_t + \chi_x x_t$ with $0 < \chi_\pi < 1$ there is indeterminacy but **no solution** is stable under learning.

2. For $i_t = \chi_\pi E_t^* \pi_{t+1} + \chi_x E_t^* x_{t+1}$ there are cases in which

   (a) noisy finite-state **Markov SSEs** are stable under learning.

   (b) "common factor" SSEs are stable under learning,

$$\begin{aligned} y_t &= a + cv_t + d\zeta_t, \text{ where } y_t' = (\pi_t, x_t) \text{ and } v_t' = (g_t, u_t). \\ \zeta_t &= \lambda \zeta_{t-1} + \varepsilon_t \end{aligned}$$

   for sunspot ζ_t, where λ satisfies a "**resonant frequency**" condition. (ζ_t generalizes finite-state Markov SSEs).

Results for optimal i_t rules (EH, REStud 2003, ScandJE 2006)

1. Fundamentals based reaction function

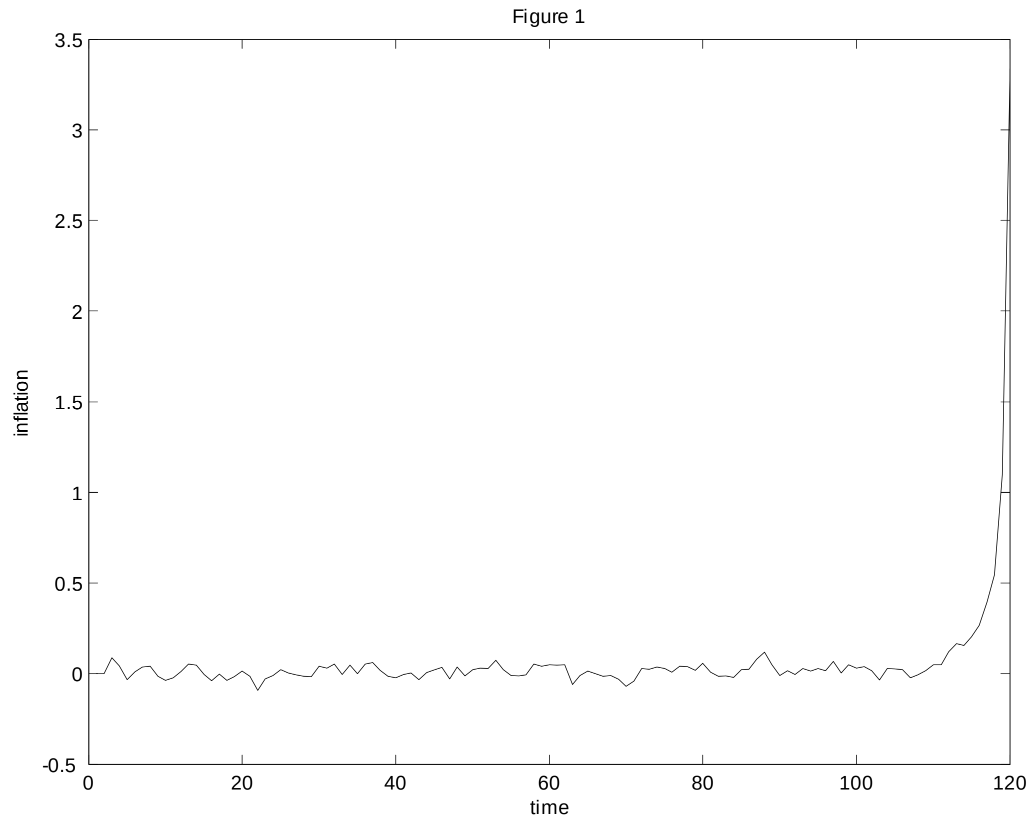
$$i_t = \psi_x x_{t-1} + \psi_g g_t + \psi_u u_t.$$

   Instability under learning and also indeterminacy can arise.

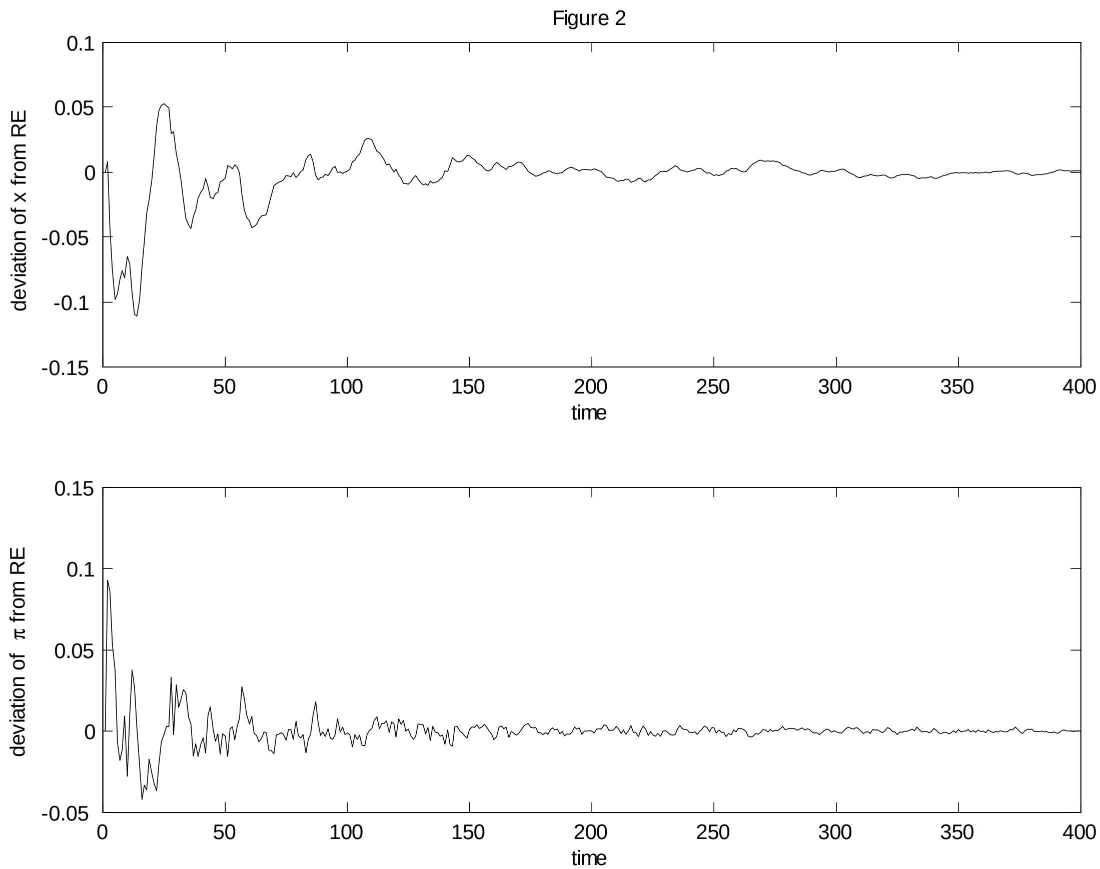
2. Expectations-based rule

$$i_t = \delta_L x_{t-1} + \delta_\pi E_t^* \pi_{t+1} + \delta_x E_t^* x_{t+1} + \delta_g g_t + \delta_u u_t$$

   with correctly chosen parameters yields an REE that is always determinate and learnable.



Instability under fundamnetals-based rule



Stability under expectations-based rule

Four Applications

(i) Monetary policy under discounted LS

Orphanides and Williams (2005a). Lucas-type aggregate supply curve for inflation π_t:

$$\pi_{t+1} = \phi\pi^e_{t+1} + (1-\phi)\pi_t + \alpha y_{t+1} + e_{t+1},$$

– Output gap y_{t+1} is set by monetary policy up to white noise control error

$$y_{t+1} = x_t + u_{t+1}.$$

– Policy objective function $\mathcal{L} = (1-\omega)Var(y) + \omega Var(\pi - \pi^*)$ gives rule

$$x_t = -\theta(\pi_t - \pi^*).$$

where under RE $\theta = \theta^P(\omega, \phi, \alpha)$.

Learning: Under RE inflation satisfies

$$\pi_t = \bar{c}_0 + \bar{c}_1 \pi_{t-1} + v_t.$$

Under learning private agents estimate c_0, c_1 by **constant gain** (**discounted**) LS ("perpetual learning")

- Discounting of data natural if agents are concerned to track structural shifts.

- There is some empirical support for constant gain learning.

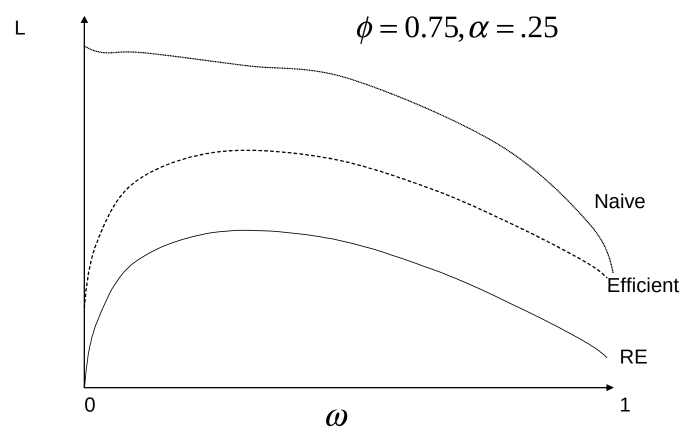
With constant gain, LS estimates fluctuate randomly around $(\bar{c}_0, \bar{c}_1)$: there is "perpetual learning" and

$$\pi^e_{t+1} = c_{0,t} + c_{1,t} \pi_t.$$

Results: – Perpetual learning increases inflation persistence.

– Naive application of RE policy leads to inefficient policy. Incorporating learning into policy response can lead to major improvement.

– Efficient policy is more hawkish, i.e. under learning policy should increase θ to reduce persistence. This helps guide expectations.



Policymaker's loss

(ii) Explaining Hyperinflations (Marcet&Nicolini AER, 2003)

Seigniorage model of inflation extended to open economies.

Basic hyperinflation model: money demand

$$M_t^d/P_t = \phi - \phi\gamma(P_{t+1}^e/P_t)$$

is combined with exogenous government purchases $d_t = d > 0$ financed by seigniorage:

$$M_t = M_{t-1} + d_t P_t$$

$$\frac{P_t}{P_{t-1}} = \frac{1 - \gamma(P_t^e/P_{t-1})}{1 - \gamma(P_{t+1}^e/P_t) - d/\phi}.$$

For $d > 0$ not too large, there are two steady states $\beta = \frac{P_t}{P_{t-1}}$, $\beta_L < \beta_H$.

Under steady state learning: agents estimate β based on past inflation:

$$(P_{t+1}/P_t)^e = \beta_t \text{ where } \beta_t = \beta_{t-1} + t^{-1}(P_{t-1}/P_{t-2} - \beta_{t-1}).$$

One can show that β_L is E-stable, while β_H is not: $\beta_t > \beta_H \to \infty$.

Hyperinflation stylized facts:

– Recurrence of hyperinflation episodes.

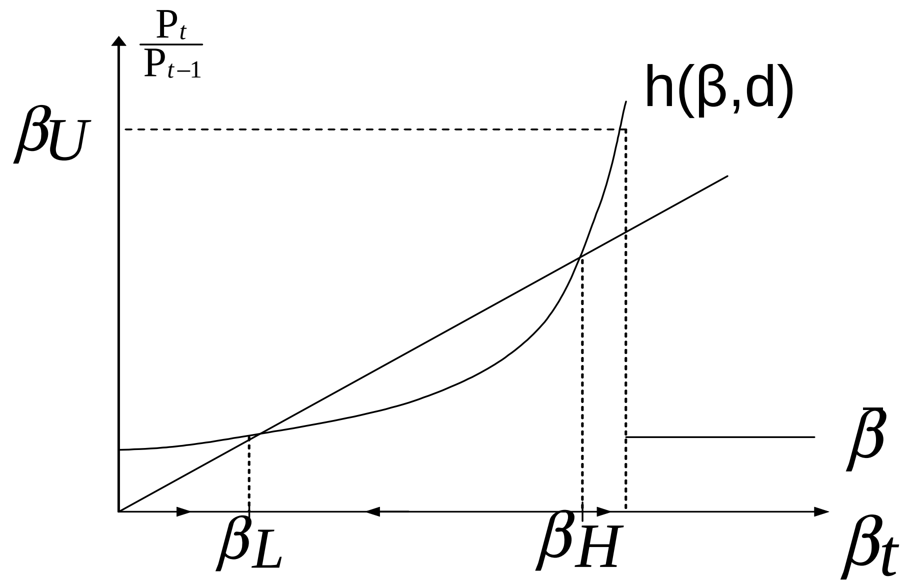
– ERR (exchange rate rules) stop hyperinflations, though new hyperinflations eventually occur.

– During a hyperinflation, seigniorage and inflation are not highly correlated.

– Hyperinflations only occur in countries where seigniorage is on average high.

Marcet-Nicolini's extension:

When $P_t/P_{t-1} > \beta^U > \beta_H$ inflation is stabilized by moving to an ERR.



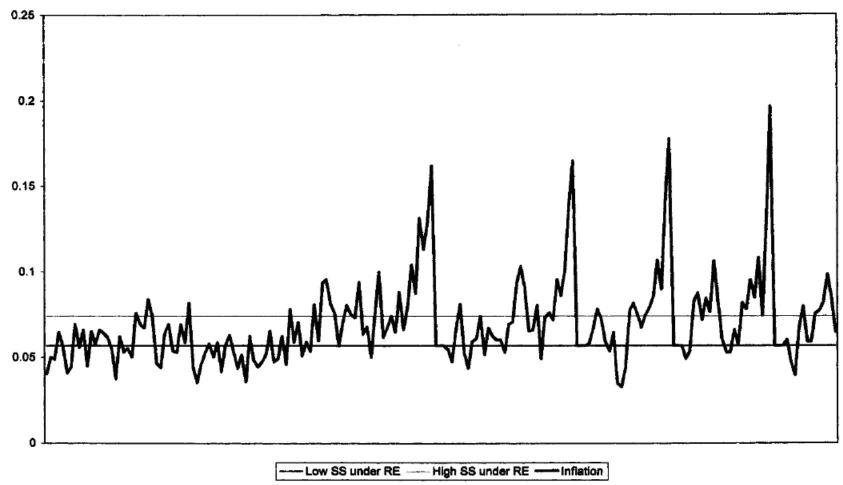
Inflation as a function of expected inflation

– The low inflation steady state is locally learnable.

– A sequence of adverse shocks can create explosive inflation.

– The learning dynamics lead to periods of stability alternating with occasional eruptions into hyperinflation.

– The learning approach can explain all the stylized facts.



Hyperinflations under learning

(iii) Learning about risk & return: bubbles and crashes

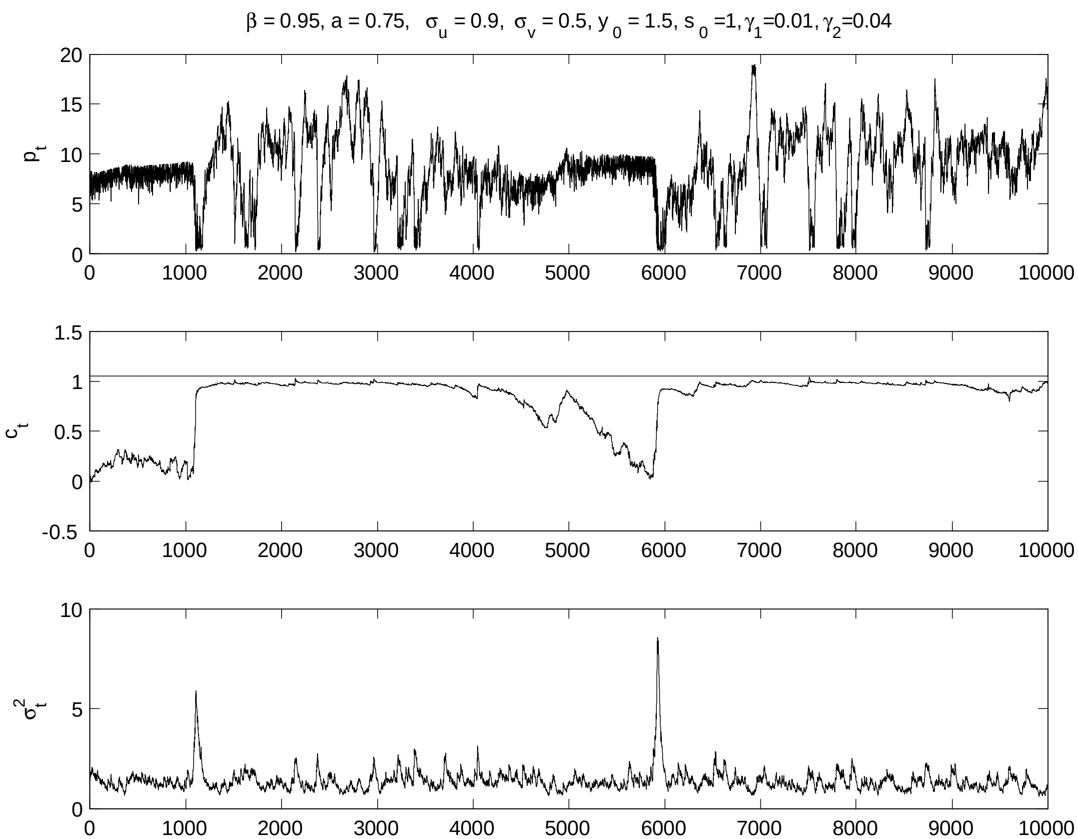
Branch and Evans (AEJ:Macro, July 2011) use a simple mean-variance linear asset pricing model. OLG set-up with 2-period planning horizons.

$$p_t = \beta E_t^* \left(p_{t+1} + y_{t+1}\right) - \beta a \sigma_t^2 z_{st}.$$

σ_t^2 is the estimate of the conditional variance of returns.

With *iid* dividend and supply shocks, the REE for p_t is a constant + white noise. Under learning, agents forecast p_t as an AR(1) using discounted LS and estimate σ_t^2 using a simple recursive algorithm.

If agents discount past data, prices under learning will occasionally break free from their fundamentals and exhibit bubbles and crashes. This results from the self-referential feature of the model.


β = 0.95, a = 0.75, σu = 0.9, σv = 0.5, y0 = 1.5, s0 =1, γ1=0.01, γ2=0.04
pt
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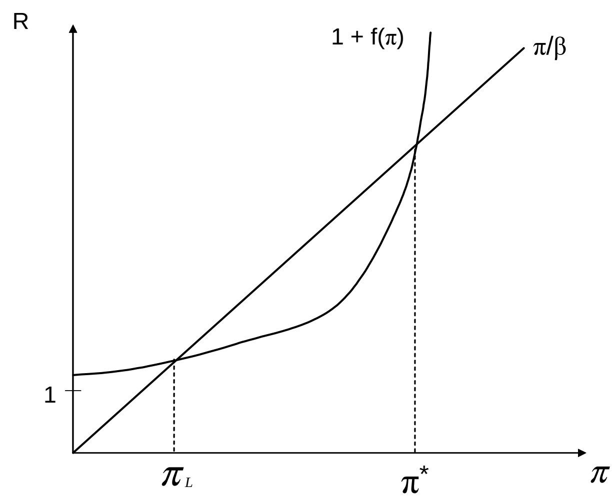
10000

(iv) Liquidity Traps, Learning & Stagnation

Evans, Guse, Honkapohja (EER, 2008), look at the liquidity traps with learning. Possibility of a "liquidity trap" under a global Taylor rule subject to zero lower bound shown by Benhabib, Schmitt-Grohe and Uribe (2001, 2002) for RE.



Multiple steady states with global Taylor rule.

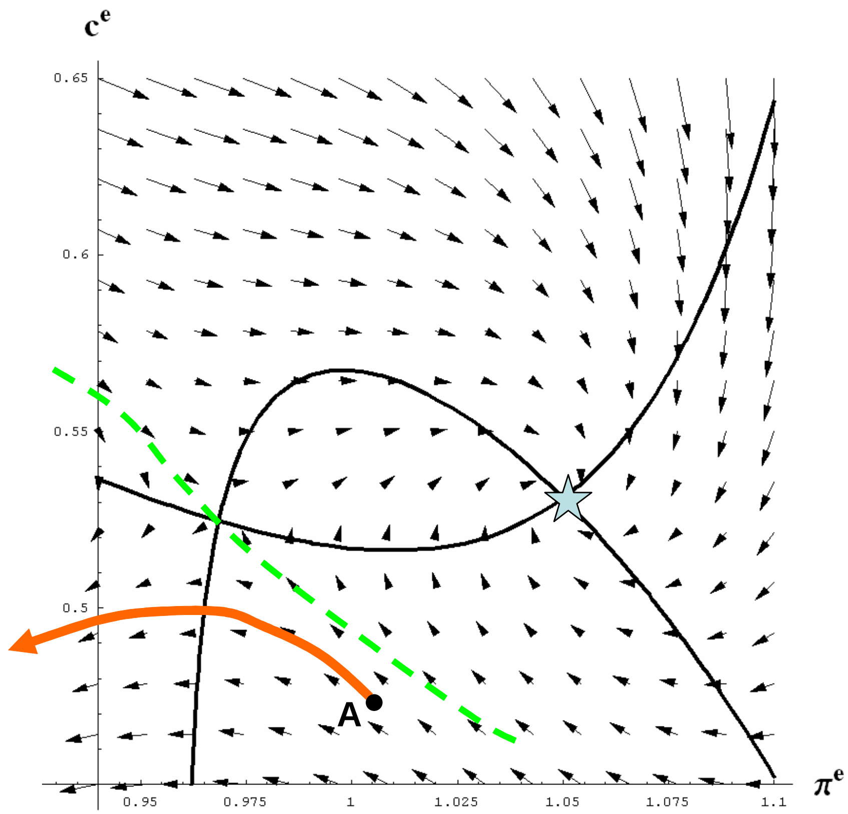
NK model with monopolistic competition, price-adjustment costs, & global Taylor-rule. Normal fiscal policy: fixed government purchases g_t and a "passive" tax policy. EGH add simple adaptive learning.

The key equations are the PC and IS curves

$$\begin{aligned} \frac{\alpha\gamma}{\nu}(\pi_t - 1)\pi_t &= \beta\frac{\alpha\gamma}{\nu}\left(\pi^e_{t+1} - 1\right)\pi^e_{t+1} \\ &\quad +(c_t + g_t)^{(1+\varepsilon)/\alpha} - \alpha\left(1 - \frac{1}{\nu}\right)(c_t + g_t)c_t^{-\sigma_1} \\ c_t &= c^e_{t+1}(\pi^e_{t+1}/\beta R_t)^{\sigma_1}, \end{aligned}$$

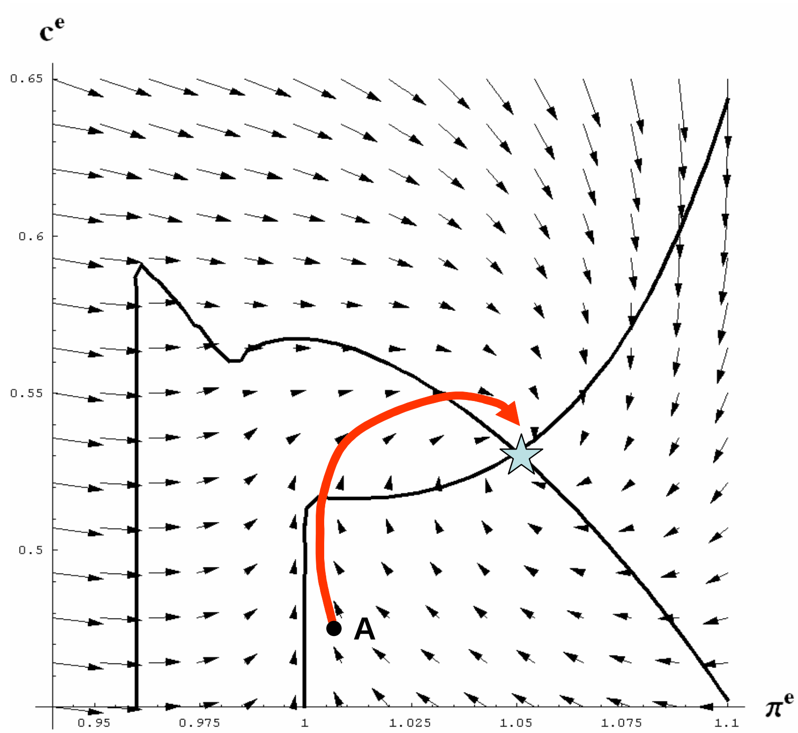
Two stochastic steady states at π_L and π^*. Under "steady-state" learning, π^* is locally stable but π_L is not.

Pessimistic expectations c^e, π^e can lead to a deflationary spiral and stagnation.



π^e and c^e dynamics under normal policy

Solution: aggressive policies at an **inflation threshold** $\pi_L < \tilde{\pi} < \pi^*$.Reduce R_t to the ZLB and if necessary increase g_t to maintain $\tilde{\pi}$.



Inflation threshold $\tilde{\pi}$, $\pi_L < \tilde{\pi} < \pi^*$, for aggressive monetary policy and, if needed, aggressive fiscal policy.

Conclusions

- Expectations play a large role in modern macroeconomics.
- Cognitive consistency principle, e.g. model agents as econometricians.
- Stability of RE under private agent learning is not automatic.
- Learning has the potential to explain various empirical phenomena difficult to explain under RE: volatility of expectations, hyperinflation, asset price bubbles, stagnation.
- Policymakers may need to use policy to guide expectations.