
Math 242
Answers for the final exam review problems

1. Let $f(x, y) = 3x^2y + x^3 + y - 2$. Find the equation for the tangent plane to the graph of f at the point $(2, 1, 3)$.

Answer: $z = 24x + 13y - 58$

2. Find and classify the critical points of the function $f(x, y) = x^3 - y^3 - 2xy$.

Answer: $(0, 0)$ is a saddle and $(-\frac{2}{3}, \frac{2}{3})$ is a local maximum.

3. You want to build a rectangular box. The material for the four sides will cost $\$0.50/\text{ft}^2$ and the material for the top and bottom will cost $\$0.75/\text{ft}^2$. If the box is to have volume 6000ft^3 , what should the dimensions be to minimize the cost?

Solution: $w = d = 15.87\text{ ft}$ and $h = 23.81\text{ ft}$.

4. Find and classify the critical points of the function $f(x, y) = 2x^2 - 6xy - y^3$.

Answer: $(0, 0)$ is a saddle and $(-\frac{9}{2}, -3)$ is a local minimum.

5. Consider the linear system

$$3x + 2y + 4z = 10$$

$$2x - y - 5z = 6$$

$$6x + y + z = 4.$$

Give the augmented matrix for this linear system, and then give the rref form of the matrix. Write down the general solution to the linear system.

Answer: $x = -1.4$, $y = 17.7$, $z = -5.3$.

6. Consider the linear system

$$2x - 4y + 3z + 29w = 4$$

$$x - 2y + 7w = 1$$

$$x - 2y + z + 12w = 1$$

$$3x - 6y - z + 16w = 2.$$

Give the augmented matrix for this linear system, and then give the rref form of the matrix. Write down the general solution to the linear system.

Answer: The system has no solution.

7. Consider the linear system

$$x - 2y - z = 1$$

$$y + 2z = 2$$

$$3x - 2y + 5z = 11$$

Give the augmented matrix for this linear system, and then give the rref form of the matrix. Write down the general solution to the linear system.

Answer: $x = 5 - 3z$, $y = 2 - 2z$, and z can be anything.