
Math 242
Answers for Midterm #2 review problems.

1. Find the average value of the function $f(x) = 3x^2 - x$ on the interval from $x = 2$ to $x = 6$.
Answer: 48.
2. Find the area of the region lying above the graph of $y = e^x$, below the graph of $y = x^2 + 2$, to the right of $x = 0$, and to the left of $x = 1$.
Answer: $\frac{10}{3} - e$.
3. Find the area of the region enclosed by $y = 2x^2 - 2$ and $y = 1 - x^2$.
Answer: 4.
4. Money is deposited into a bank account in a continuous stream, at a rate of \$1,000 per month. The account earns an annual interest rate of 6% every year, compounded continuously. How much money will be in the account at the end of 5 years?
Answer: \$69,971.76.

Solution: We are depositing money at a rate of \$12,000 per year. The formula for future value of a continuous income stream gives us

$$\int_0^5 12,000 \cdot e^{0.06(5-t)} dt.$$

Remember that $e^{0.06(5-t)} = e^{0.3-0.06t} = e^{0.3} \cdot e^{-0.06t}$. So we are looking at

$$\begin{aligned} \int_0^5 12,000 \cdot e^{0.06(5-t)} dt &= \int_0^5 12,000 \cdot e^{0.3} \cdot e^{-0.06t} dt = 12,000e^{0.3} \cdot \left[\frac{e^{-0.06t}}{-0.06} \right]_0^5 dt \\ &= -\frac{12,000}{0.06} \cdot e^{0.3} \cdot [e^{-0.3} - e^0] \approx \$69,971.76. \end{aligned}$$

5. Determine which of the following integrals converge and which diverge. For the ones that converge, compute the integral.

$$(a) \int_1^\infty \frac{5}{x^2} dx \quad (b) \int_0^1 \frac{1}{x^3} dx \quad (c) \int_2^\infty e^{-3x} dx \quad (d) \int_9^\infty \frac{1}{\sqrt{x}} dx.$$

Answer: Only (a) and (c) converge. The answer for (a) is 5 and the answer for (c) is $\frac{e^{-6}}{3}$.

6. Consider the differential equation $\frac{dy}{dx} = (x+1)y$.
 - (a) Give the general solution.
 - (b) Give the particular solution satisfying $y = 5$ when $x = 0$.Answer: (a) $y = De^{(\frac{x^2}{2}+x)}$ (b) $y = 5e^{(\frac{x^2}{2}+x)}$.

7. A certain company's yearly revenue is projected to be $R(t) = 52(0.8)^t$ million dollars, where t is the number of years after 2005.

- (a) Find the company's average yearly revenue over the next ten years.
 (b) If the company lives into the indefinite future, what will their total revenue be from 2005 on?
 Answer: (a) 20.8 million dollars (b) 233.03 million dollars.

Solution to (a): The average value is

$$\frac{1}{10-0} \int_0^{10} 52(0.8)^t dt = \frac{1}{10} \int_0^{10} 52e^{\ln(0.8)t} dt = \frac{52}{10} \cdot \frac{1}{\ln(0.8)} e^{\ln(0.8)t} \Big|_0^{10} = \frac{52}{10 \ln(0.8)} [e^{10 \ln(0.8)} - e^0] \approx 20.8$$

and remember that $R(t)$ is measured in millions of dollars.

8. If $f(x, y) = 3x^2 - 2y^5 + x^2y - x$, compute the following quantities:

(a) $\frac{\partial f}{\partial x}$ (b) $\frac{\partial f}{\partial y}$ (c) $\frac{\partial f}{\partial x} \Big|_{(2,1)}$ (d) $\frac{\partial f}{\partial y} \Big|_{(1,3)}$.

Answer: (a) $6x + 2xy - 1$ (b) $-10y^4 + x^2$ (c) 15 (d) -809.

9. (a) Compute the integral $\int \frac{2x}{x^2 + 1} dx$.

(b) Solve the differential equation $\frac{dy}{dx} = \frac{2xy}{x^2 + 1}$ subject to the initial condition $y = 9$ when $x = 0$.

Answer: (a) $\ln|x^2 + 1| + C$. (b) $y = 9x^2 + 9$.