Math 242
Answers for Midterm #1 review problems

1. Evaluate the following integrals:

$$(a)\quad \int (2-3x+x^4)dx \qquad (b)\quad \int \left(3e^x + \frac{2}{x} - \sqrt{x}\right)dx \qquad (c)\quad \int \frac{3x}{(x^2+1)^3}dx.$$

Answers:
(a) $2x - \frac{3}{2}x^2 + \frac{1}{5}x^5 + C$.
(b) $3e^x + 2\ln|x| - \frac{2}{3}x^{3/2} + C$
(c) $-\frac{3}{4}(x^2+1)^{-2} + C$.

2. Evaluate the following integrals:

$$(a)\quad \int_0^2 4e^{3x}dx \qquad (b)\quad \int_{-1}^1 \left(t^2 - \frac{2}{t^3}\right)dt \qquad (c)\quad \int_1^2 \left(z^2 + 5z^2e^{z^3}\right)dz$$

Answers:
(a) $\frac{4}{3}(e^6-1)$.
(b) $\frac{2}{3}$
(c) $\frac{7}{3} + \frac{5}{3}e^8 - \frac{5}{3}e^1$.

3. The marginal cost of producing the xth box of widgets is $50 + 3x + \frac{1}{x}$. The total cost of producing one box is \$100. Find the cost of producing the first x boxes.

Answer: $C(x) = 50x + \frac{3}{2}x^2 + \ln|x| + \frac{97}{2}$.

4. In a certain colony of insects, the rate at which the colony is growing after t years of our study is given by $90t^2 - 50t + 10$ (measured in insects per year). If the insect population was 30 after the first year of our study, find the population after year t. What was the population after year 3?

Answer: $P(t) = 30t^3 - 25t^2 + 10t + 15$. And $P(3) = 630$.

Solution: Let $P(t)$ denote the population after year t. We are told $P'(t) = 90t^2 - 50t + 10$, and also that $P(1) = 30$. Then

$$P(t) = \int P'(t)dt = \int (90t^2 - 50t + 10)\,dt = 30t^3 - 25t^2 + 10t + C.$$

Plugging in $t = 1$ gives

$$30 = P(1) = 30 \cdot 1^3 - 25 \cdot 1^2 + 10 \cdot 1 + C = 15 + C.$$

Then by algebra, $C = 15$.

So $P(t) = 30t^3 - 25t^2 + 10t + 15$. To find the population after year 3, plug in $t = 3$.

5. Compute the area of the region that is contained beneath the graph of $y = \frac{3x}{x^2+2}$, above the x-axis, to the right of the line $x = 3$, and to the left of the line $x = 6$

   Answer: $\frac{3}{2}\Big(\ln(38) - \ln(11)\Big)$.

6. Evaluate the following integrals:

$$(a) \quad \int_0^2 (6x+3)(x^2+x)^4\, dx \qquad (b) \quad \int \frac{2x+1}{x^2+x}\, dx \qquad (c) \quad \int_1^2 \frac{2x+1}{x+2}\, dx$$

   Answer:

   (a) $\frac{3}{5} \cdot 6^5$.

   (b) $\ln|x^2+x| + C$.

   (c) $2 + 3(\ln 3 - \ln 4)$.

7. The marginal cost for producing the xth box of frisbees is $x^2(10+x)$. The fixed cost is \$500 (the fixed cost is the cost of producing 0 boxes). Using this information, determine the cost of producing the first x boxes.

   Answer: $C(x) = \frac{10}{3}x^3 + \frac{1}{4}x^4 + 500$.

8. Compute $\int (6x+1)e^{3x^2+x}\, dx$.

   Answer: $e^{3x^2+x} + C$.

9. If I tell you that $\frac{d}{dx}\left(\frac{1}{\sqrt{4+x^2}}\right) = \ln(x + \sqrt{4+x^2})$, use this information to evaluate

$$\int_0^2 \frac{1}{\sqrt{4+x^2}}\, dx.$$

   Answer: $\ln(2+\sqrt{8}) - \ln 2$.

   Solution: Remember that if $F'(x) = f(x)$, then $\int_a^b f(x) = F(b) - F(a)$. In our case $f(x) = \frac{1}{\sqrt{4+x^2}}$ and $F(x) = \ln(x + \sqrt{4+x^2})$. So

$$\begin{aligned} \int_0^2 \frac{1}{\sqrt{4+x^2}} &= \ln(x+\sqrt{4+x^2})\Big]_0^2 \\ &= \ln(2+\sqrt{4+4}) - \ln(0+\sqrt{4+0}) \\ &= \ln(2+\sqrt{8}) - \ln(2). \end{aligned}$$